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Review of power curve modelling for wind turbines

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ABSTRACT

Currently, variable speed wind turbine generators (VSWTs) are the type of wind turbines most widely installed. For wind energy studies, they are usually modelled by means the approximation of the manufacturer power curve using a generic equation. In literature, several expressions to do this approximation can be found; nevertheless, there is not much information about which is the most appropriate to represent the energy produced by a VSWT. For this reason, in this paper, it is carried out a review of the equations commonly used to represent the power curves of VSWTs: polynomial power curve, exponential power curve, cubic power curve and approximate cubic power curve. They have been compared to manufacturer power curves by using the coefficients of determination, as fitness indicators, and by using the estimation of energy production. Data gathered from nearly 200 commercial VSWTs, ranging from 225 to 7500 kW, has been used for this analysis. Results of the analysis presented in the paper show that exponential and cubic approximations give the higher R^2 values and the lower error in energy estimation. With the approximate cubic power curve quite high values of $R²$ and low errors in energy estimation are achieved, which makes this kind of approximation very interesting due to its simplicity. Finally, the polynomial power curve shows the worst results mainly due to its sensitivity to the data given by the manufacturer.

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Abbreviations: WTG, wind turbine generator; VSWT, variable speed wind turbine $*$ Corresponding author. Tel.: $+34$ 986 813 912.

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1. Introduction

The power curve of a WTG is obtained by the manufacturers from field measurements of wind speed and power, apart from environmental values (temperature, pressure and relative

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humidity). The measurements are usually averaged and normalised to a reference air density using normalised procedures [1]. The resulting discrete values of the power curve for a determined WTG are usually available from manufacturers, and they can be used for studies involving energy evaluation.

Nevertheless, for the sake of generality, it is common that a generic equation for modelling the power curve will be preferred in studies about WTG modelling [2–5], analysis of wind energy potential [6], site matching [5,7–9], cost modelling [10,11], etc. In this context, the use of an equation for representing a power curve and the obtention of its parameters becomes an important issue. The main problem derived from using a generic equation is the fact that is hard to know how this equation will accurately represent any commercial WTG.

In the first term, the power curve of a WTG can be estimated using the power curve coefficient (C_p) from the turbine blade parameters (blade design, tip speed ratio and pitch angle) [4], the rotor dimensions and the reference air density. For example in [12] the power coefficient is calculated through an expression that links the blade radius, blade design constant and wind turbine shaft angular speed with the power coefficient. In [11] an expression is proposed for the approximation of C_p , considering a rated power coefficient, rated wind speed and a parameter expressing the operation range of wind speed. The shortcomings of using the models proposed in [11] and [12] are that they depend on some technical factors of the wind turbines which are difficult to obtain from the manufacturers.

Another way to approximate the power curve is presented in [2], where power curves are approximated by means of fitting techniques, like least squares or cubic spline interpolation. Although pretty accurate fits are achieved, the resulting power curve equations are quite complex, which makes it difficult to find a generic expression.

To overcome the problems depicted above, the power curve of WTGs is usually represented by means of a polynomial power curve [13–15] or by means of an exponential power curve or its simplifications [16]. Their parameters can be derived from manufacturer data or by fitting the manufacturer power curve. However, although these expressions are widely used, there is little evidence of how these curves fit with real WTGs [13–18]. For this reason, in this paper is presented a study of the power curve models taking into account a database with manufacturer information from nearly 200 variable speed wind turbines (VSWT). Only VSWTs have been considered in this paper because they represent the state of art of commercial WTGs installed at present. The most important wind turbine manufacturers have been included in this database.

In order to analyse which are the most appropriate equations to approximate power curves, it is also presented a critical comparison of the fitted power curves considering the coefficient of determination R^2 , as a measure of goodness of fit, and the difference between the estimations of energy density when the fitted and the manufacturer power curves are used.

The paper is organised as follows. Section 2 presents the identification of the main features of the power curve. Section 3 summarises the most typical models used for the representation of the power curves. Section 4 shows the database used for the characterisation of the power curves including the main characteristics of the wind turbines. In Section 5, the results of the fitting methods and indicators of fitness are presented. Finally, conclusions are given in Section 7.

2. Energy evaluation and power curve

The available power of the wind that crosses the rotor of a wind turbine can be obtained from

$$
p_w(v) = \frac{1}{2}A\rho v^3\tag{1}
$$

where $p_w(v)$ is the power in W associated to a wind speed v in m/s, A is the rotor area in m^2 and ρ is the air density (typ. 1225 kg/m³) [1]). This power is related to power generated by a wind turbine by means of the power coefficient

$$
C_p(v) = p(v)/p_w(v)
$$
 (2)

where $p(v)$ is the power generated by the wind turbine in W, C_p is the power coefficient that is related to the blade design, the tip angle and the relationship between rotor speed and wind speed. The maximum theoretical value of power coefficient, known as the Betz limit, is 0.593 (16/27). However, this value is not achievable with real turbines and its maximum value is normally around 0.5. The power coefficient can be obtained from the manufacturer data, as a consequence, mechanical and electrical losses are usually included in the coefficient value as well as the aerodynamic behaviour of blades.

The power delivered by a wind turbine is usually represented through its power curve, where a relation between the wind speed and the power is established. For the VSWTs, this relationship can be expressed in the following way:

$$
p(v) = \begin{cases} 0 & v < v_{ci} \text{ or } v > v_{co} \\ q(v) & v_{ci} \le v < v_{r} \\ P_{r} & v_{r} \le v \le v_{co} \end{cases}
$$
(3)

where $p(v)$ is the electric power in W, v_{ci} is the cut-in wind speed in m/s, v_{co} is the cut-out wind speed in m/s, v_r is the rated wind speed in m/s, P_r is the rated power in W and $q(v)$ is the non-linear relationship between power and wind speed (see Fig. 1).

The shape of the non-linear part is related to the control strategy of extracting as much power as possible from the wind. This is why it is roughly represented by a cubic expression [17].

The zones of the power curve defined by cut-in, rated and cut-out wind speeds are clearly specified in (3). Nevertheless, it must be kept in mind that the power curve is obtained from mean values of a set of measurements [1]. This is the main explanation for the typical smooth shape of the power curve. Consequently, the limits shown in (3) are not as clearly defined in manufacturer power curve as those shown in the mentioned equation.

The energy density E in W/m^2 for a specific wind site and a wind turbine can be obtained by using the power curve and the probability distribution function of wind speed

$$
E = \frac{1}{A} \int_{v_{ci}}^{v_{co}} p(v) f(v) dv
$$
 (4)

where $f(v)$ represents the probability in p.u. associated to the wind speed ν [19]. The discrete version of this equation can be written as

$$
E = \frac{1}{A} \sum_{j=1}^{N} fr_j p(v_j) = \frac{1}{A} \sum_{j=1}^{R-1} fr_j p(v_j) + \frac{1}{A} Pr \sum_{j=R}^{O} fr_j
$$
(5)

where N is the number of power curve values, $v_I = v_{ci}$ is the cut-in speed, $v_0 = v_{\rm co}$ is the cut-out wind speed, $v_R = v_r$ is the rated wind speed and f_{ri} is the relative frequency associated to each wind speed v_i that can be obtained from the histogram of wind speeds.

3. Power curve characterisation

The most typical mathematical equations for representing the non-linear part $q(v)$ of a power curve are:

- Polynomial power curve.
- Exponential power curve.
- Cubic power curve.
- Approximate cubic power curve.

3.1. Polynomial power curve

In the polynomial power curve approximation, a second degree polynomial is used to fit $q(v)$ [13–15]

$$
q(v) = C_1 + C_2 v + C_3 v^2
$$
 (6)

where C_1 , C_2 and C_3 are coefficients calculated from v_{ci} , P_r and v_r .

Fig. 1. Representation of the power curve.

$$
C_2 = \frac{1}{(\nu_{ci} - \nu_r)^2} \left[4(\nu_{ci} + \nu_r) \left(\frac{\nu_{ci} + \nu_r}{2\nu_r} \right)^3 - 3\nu_{ci} - \nu_r \right]
$$

$$
C_3 = \frac{1}{(\nu_{ci} - \nu_r)^2} \left[2 - 4 \left(\frac{\nu_{ci} + \nu_r}{2\nu_r} \right)^3 \right]
$$

3.2. Exponential power curve

When an exponential power curve is used to model a VSWT power curve, the non-linear curve $q(v)$ is approximated by using [16]

$$
q(\nu) = \frac{1}{2}\rho A K_p \left(\nu^{\beta} - \nu_{ci}^{\beta}\right) \tag{7}
$$

where K_p and β are constants.

3.3. Cubic power curve

A typical simplification of the expression shown in (7) can be obtained supposing v_{ci} equal to zero and β equal to three. As a result, a cubic power curve approximation, that is similar to (1), is obtained [5,20]

$$
q(\nu) = \frac{1}{2}\rho A C_{p,eq} \nu^3
$$
\n⁽⁸⁾

where $C_{p,eq}$ is a constant equivalent to the power coefficient.

3.4. Approximate cubic power curve

An approximation of (8), called approximate cubic power curve, can be obtained by assuming $C_{p,eq}$ equal to the maximum value of effective power coefficient ($C_{p,max}$). The term "effective" means that mechanical and electrical losses are included in this coefficient. The resulting equation is

$$
q(v) = \frac{1}{2}\rho A C_{p,\text{max}} v^3
$$
\n(9)

4. Wind turbine characteristics

The first step in order to compare the power curves is to gather VSWT data from different WTG databases and information from manufacturers [21–26]. A database of 187 VSWTs, including parameters like power curve data and type of generator, has been used (see Appendix B). As an example, the representation of the power curves can be seen in Fig. 2. The values of cut-in, cut-out, rated wind

Fig. 2. Representation of all power curves in database.

speed, maximum hub height and the type of generator are presented in Figs. 3–5. Finally, the VSWT technologies are shown in Fig. 6.

As a first analysis, it can be concluded that the typical values for cut-in wind speeds are lower than 5 m/s (v_{ci} < 5 m/s), for cut-out wind speeds are higher than 15 m/s ($v_{\rm co} > 15$ m/s), and rated wind speeds lie between 8 m/s and 18 m/s (8 m/s $\lt v_{cr}$ \lt 18 m/s) as can be seen in Fig. 5.

Aiming to guarantee the consistency of data, v_{ci} , v_r and v_{co} have been directly obtained from the manufacturer power curve.

5. Power curve modelling

5.1. Power curve fitting

The main objective of this paper is to determine which of the equations presented in Section 3 are the most appropriate to represent the behaviour of the power curves given by the manufacturers. For this reason the proposed equations to be evaluated are

Fig. 3. Histogram of rated power.

Fig. 4. Histogram of maximum hub height.

Fig. 5. Histogram of cut-in, rated and cut-out wind speeds.

the polynomial (6), the exponential (7), the cubic (8) and the approximate cubic (9) power curves. The parameters for the calculation of polynomial power curve, C_1 , C_2 and C_3 in (6), and the parameter of approximate cubic power curve, $C_{p,max}$ in (9), can be obtained directly from the manufacturer data.

In the other hand, the parameters of the exponential power curve, K_p and β in (7), and the parameter in the cubic power curve, $C_{p,eq}$ in (8), must be calculated using a curve fitting method. In this case, it has been used a least squares one which minimises the following index:

$$
J = \sum_{j=1}^{R-1} (q(v_j) - q'(v_j))^2
$$
\n(10)

where $q(v)$ represents the non-linear part of manufacturer power curve (see Fig. 1) and $q'(v)$ is its corresponding fitted curve.

The index J has been minimised using a Nelder–Mead simplex method implemented in MATLAB [27].

5.2. Goodness of fit

Two indicators of goodness of fit have been selected for the comparison of the power curves: the coefficient of determination $R²$ and the mean energy production calculated by using different mean wind speeds.

5.2.1. Coefficient of determination \mathbb{R}^2

The coefficient of determination R^2 is used to compare the results of the manufacturer power curve with the power curves

Fig. 6. Histogram of VSWT technologies.

Fig. 7. Rayleigh probability density function of the mean wind speeds (5, 6, 7, and 8 m/s

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obtained with (6)–(9). This coefficient can be defined as

$$
R^{2} = 1 - \frac{\sum_{j=1}^{R-1} (q(v_{j}) - q'(v_{j}))^{2}}{\sum_{j=1}^{R-1} (q(v_{j}) - \xi(q(v_{j})))^{2}}
$$
(11)

where $\zeta\{q(v)\}\$ is the mean of the non-linear part of the manufacturer power curve.

In this case, the R^2 coefficient is closely related to the expression of index J in (10) used during the curve fitting process. Thus, for the exponential and cubic power curves the R^2 values are supposed to be the highest.

5.2.2. Energy production

Another selection criteria considered, in order to determine which is the most appropriate equation that fits the manufacturer power curves, is the relative error between the energy calculated from manufacturer power curve and the energy obtained from the fitted power curves. For this calculation, a set of Rayleigh PDF with different mean wind speeds $(5, 6, 7, 2, 8, 8, 7)$ has been used. Its representation can be seen in Fig. 7 [19]. These mean wind speeds have been chosen because they are typical in wind energy installations, and with them can be achieved the common utilisation times between 2200 and 3500 h/year [17].

The error of energy density ε in % is calculated by means of the following expression:

$$
\varepsilon = \frac{E'-E}{E} \times 100\tag{12}
$$

where E is the energy density obtained from manufacturer power curve and E' is the energy density obtained from the fitted power

Table 1

Fig. 8. Power curve and its approximations.

Parameters of approximation equations for a 2 MW VSWT.

Table 3

Goodness of fit for a 2 MW VSWT.

Fig. 9. Values obtained for the coefficient of determination R^2 .

Table 4

Summary of R^2 values: mean and standard deviation.

curve (polynomial, exponential, cubic or approximate cubic power curve).

6. Results

All equations for power curve modelling have been applied for each power curve in the database. As an example, a power curve of a 2 MW VSWT, whose manufacturer power curve is shown in Table 1, has been analysed. The approximations curves are presented in Fig. 8 and its parameters can be seen in Table 2. For this case, the obtained results are shown in Table 3.

Fig. 9 represents the R^2 results from fitting the power curves of all the VSWTs from the database. As can be seen, the R^2 values for the polynomial fitting are the worst with values lower than 0.5. This is because the polynomial expression is the one that depends the most on data presented by the manufacturer in the power curve. Also, it can be seen in Fig. 9 that R^2 values for the exponential, cubic and the approximate cubic are over the 0.92 which is a pretty good fit. So far, it can be concluded that the exponential and the cubic have the best behaviour. Table 4 shows a summary of the mean and standard deviation values of the results obtained for R^2 .

The distributions of errors of energy density ε for the polynomial, exponential, cubic and the approximate cubic power curves are shown in Fig. 10. The main conclusion here is that the exponential and the cubic power curves, represented by (7) and (8), have the best behaviour in terms of mean power error and the lowest standard deviation. Results can be seen in Table 5.

Polynomial approximation has the worst results for all goodness of fit indicators. This can be explained by its strong dependence on the power curve parameters: cut-in, cut-out and rated wind speed, specially the last one. The effect that the rated wind speed value has in the fitting results is analysed in Appendix A.

7. Conclusions

A review of the most common equations (polynomial, exponential, cubic and approximate cubic) used to model VSWT power curves has been presented. They have been analysed in order to establish

Fig. 10. Distribution of mean power error for the polynomial, exponential, cubic and cubic approx. expressions at different mean wind speeds (5, 6, 7, and 8 m/s). their capability to represent commercial VSWTs. For this purpose, data from nearly 200 commercial VSWTs has been used. The comparison between power curve models has been done using the well-known coefficient of determination R^2 . Furthermore, for comparison purposes, it has been introduced the difference between the estimation of the energy production using manufactured power curve and using its approximation. For the sake of simplicity, a set reduced set of Rayleigh wind distributions has been considered for these energy estimations.

Finally, the results of evaluating the power curve modelling methods can be summarised as follows:

 Exponential and cubic equations are the best when the coefficient of determination and the error in energy density are considered.

Fig. 11. Histogram of differences between rated wind speed and modified rated wind speed (v_r-v_{rm}) .

Table 6

Summary of R^2 values with v_{rm} .

Approximation	Mean R^2	STD R ²
Polynomial	0.9820	0.0117

Table 7

Summary of ε values with v_{rm} .

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Table 8 Wind turbine database.

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Table 8 (continued)

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Table 8 (continued)

- Approximate cubic equation gives acceptable values of goodness of fit which makes this approximation very attractive due to its simplicity.
- Polynomial equation, in spite of how simply are obtained its parameters, gives the worst results in terms of fitting. This is caused by its sensitivity to the values of the parameters, especially to the rated wind speed value.

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Appendix A. The effect of rated wind speed value in polynomial approximation

The polynomial approximation is calculated by using cut-in, cutout and rated wind speeds. Its behaviour, in terms of fitting, strongly depends on the values of these parameters, especially on rated wind speed. In order to analyse this behaviour a new value for the rated wind speed called modified rated wind speed $(v_{rm}$) has been computed. v_{rm} has been calculated so that its value minimises the index J (see (10)). The histogram of differences between the rated wind speed and the modified rated wind speed is shown in Fig. 11.

Results in Tables 6 and 7 show that the values of indicators of goodness of fit for polynomial approximation have improved when v_{rm} has been considered. However this improvement in the indicators is not enough to achieve the values shown for the rest of approximations (see Tables 4 and 5). Moreover, the main advantage of polynomial approximation is how simply its parameters are obtained. Nevertheless, that simplicity disappears when the parameter v_{rm} needs to be calculated.

For the other approximation equations, the improvement achieved by using a similar procedure is negligible.

Appendix B. Wind turbine database

Table 8 shows the main values of the database of wind turbines.

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