Optimal Planning of Unbalanced Networks Using Dynamic Programming Optimization

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***Abstract*—This paper presents a method based on dynamic programming optimization to design distribution and industrial networks with unbalanced customers. Low-voltage and medium-voltage network customers can be three-phase (i.e., industrial and commercial) and single-phase (e.g,. residential and rural areas). Moreover, American medium-voltage distribution networks have single-phase lines (rural areas) and three-phase lines (urban and primary feeders). Optimization of an unbalanced network implies optimal assignation of single-phase customers to each phase. This works considers different conductor types, tapering, power losses in lines, capacity and voltage drop constraints and deterministic loads, all employing single-phase and three-phase lines. The necessary modifications to apply it by single-wire earth return systems is presented.**

***Index Terms*—Distribution network planning, dynamic programming, industrial network, network design, single-phase customers, single-wire earth return system (SWER), three-phase.**

I. Introduction

DIFFERENT planning practices and processes are needed to design networks with single-phase and three-phase customers. Networks can have single-phase lines or three-phase lines (three-wire, four-wire or five-wire) and currents can be unbalanced. Optimal network planning must take these customer characteristics into account.

The general problem in low-voltage (lv) distribution network planning is essentially to search for a radial network with lowest overall cost by taking into account: mv/lv substations (size and location), assignation of single-phase customers to the different phases, lv lines (routes and capacities) to supply a given spatial distribution of forecast loads, thermal limits (lines and substations) and voltage level.

Industrial networks have radial architecture and source location is defined, but conductor types (section and number of phases) are unknown. The problem is similar for medium-voltage (mv) distribution networks with single-phase customers (mv/lv substations).

All authors consider balanced three-phase networks, without assignation of customers to phases [1]–[21]. In general, the methods proposed by various authors assume that lv networks are always balanced three-phase [1]–[7] and similarly for mv networks [7]–[21].

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In [5], an algorithm to design lv distribution networks by using dynamic optimization is presented. This method evaluates trees, each fed by one mv/lv substation. The conductor types and locations of mv/lv substations are obtained by applying a second dynamic programming optimization algorithm. In [6] an algorithm to design an lv distribution network by using evolution strategies is presented. The algorithm to obtain conductor types and locations of mv/lv substations is the same programming optimization algorithm as in [5].

In this paper, a new method based on dynamic programming optimization is presented. The algorithm is applied in radial distribution networks. This method is an improvement on that proposed in [5], as it considers unbalanced loads in distribution networks.

The proposed algorithm is applied to a generic unbalanced lv distribution network, but it can be extended for industrial and mv networks.

The method begins with a specific tree with the loads in the nodes, which will be connected by the branches. The branches represent the paths of the lines and the mv/lv substation location will be at a node of the tree, obtained by the algorithm.

The proposed method can be implemented in the different algorithms given in [5]–[7], for evaluation of unbalanced networks made up of multiple trees.

II. Low-Voltage Network Model

This section describes the algorithm, based on dynamic programming optimization, used to obtain the conductor types (section and phases) of the lines and the location of mv/lv substation from a generic tree. The customers of the tree are fed by only one mv/lv substation.

The first step of the method begins with a dynamic programming optimization algorithm. This algorithm obtains the optimal solution of the network without voltage drop constraints. When the voltage drops of the solution are greater than the voltage drop limit, a new dynamic programming optimization algorithm is employed. This second algorithm searches for a new solution that fulfils the voltage drop constraints (steps D and E). The solution of the first algorithm is an upper limit and will be used to search new best solutions with the second algorithm.

From a generic tree T and a branch (i, j), the subtree $A_{(i,j)}$ is defined as the subtree that contains the node i, result of removing the node j. So, the $A_{(i,j)}$ subtree includes node i and branch (i, j), but does not include node j. A subtree $A_{(i,j)}$ that is fed from a node that did not belong to it, has a power $P_{A(j,i)}$ that is considered to have constant value, regardless of the distribution of the currents through the phases.

By using subtree $A_{(i,j)}$, costs can be represented as a function of currents for the various conductor types employed along each branch. According to the dynamic programming process the cost function is associated with currents in each phase of the branch (i,j) toward to the subtree $A_{(i,j)}$.

The proposed method is divided into six steps. Step A presents the algorithm to obtain the unbalanced subtree cost, without voltage drop constraints. Step B presents the method to obtain the voltage drop of a subtree. Step C presents the method to obtain the cost of an unbalanced network without voltage drop constraints. Step D explains the algorithm to calculate the cost of an unbalanced network with voltage drop constraints. Step E presents the method to determine the optimal unbalanced network with voltage drop constraints. The particular case of networks with different voltage drop constraints by the customers is explained in step F.

A. Subtree Cost

Mathematically, the cost function of subtree $A_{(i,j)}$ can be noted by $D_{(i,j)}[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}]$, where $I_{(j,i)-a}$, $I_{(j,i)-b}$, and $I_{(j,i)-c}$ are the currents in each phase of the branch (i,j) toward to the subtree $A_{(i,j)}$. Recursively the expression is

$$\begin{aligned} &D_{(i,j)}\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right] \\ &= \min_{\substack{t_k \in Y \\ I_{i-a}, I_{i-b}, I_{i-c}}} \left\{ C_{(i,j),t_k}\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right]\right\} \\ &\quad + \sum_{\substack{x=adj(i)\\ x\neq j}} \left(D_{(x,i)}\left[I_{(i,x)-a}, I_{(i,x)-b}, I_{(i,x)-c}\right]\right) \Big/ \\ &\quad \Big/ \begin{cases} I_{(j,i)-a} = I_{i-a} + \sum\limits_{\substack{x=adj(i)\\ x\neq j}} I_{(i,x)-a} \\ I_{(j,i)-b} = I_{i-b} + \sum\limits_{\substack{x=adj(i)\\ x\neq j}} I_{(i,x)-b} \\ I_{(j,i)-c} = I_{i-c} + \sum\limits_{\substack{x=adj(i)\\ x\neq j}} I_{(i,x)-c} \\ I_{i-a} + I_{i-b} + I_{i-c} = \frac{P_i}{U\cdot\cos(\varphi)} \end{cases} \end{aligned} \tag{1}$$

where

$adj(i)$	the adjacent nodes to i;
Y	set of conductor types;
t_k	k conductor type;
$C_{(i,j),t_k}$	cost of branch (i,j) with the t_k conductor type, defined by (2);
$I_{(j,i)-a}$, $I_{(j,i)-b}$, $I_{(j,i)-c}$	flow currents in branch (i,j), from node j to node i, through phases a, b, and c;
$I_{(j,i)-n}$	flow of currents in branch (i,j), in neutral conductor, defined by (4);
$I_{i-a}, I_{i-b}, I_{i-c}$	injection currents in node i, through phases a, b, and c.

The value of $I_{(j,i)-n}$, is defined by the expression

$$\begin{aligned} I_{(j,i)-n} &= \mathrm{mod}\left\{\mathcal{I}_{(j,i)-n}\right\} \\ \mathcal{I}_{(j,i)-n} &= \frac{I_{(j,i)-a} + a^2\cdot I_{(j,i)-b} + a\cdot I_{(j,i)-c}}{a} = 1_{\underline{|120^\circ}}. \end{aligned} \tag{2}$$

Three-Phases Line Cost: The three-phase line cost of the branch (i,j) is defined by the expression

$$\begin{aligned} &C_{(i,j),t_k}\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right] \\ &= \left(c_{t_k} + c'_{t_k}\cdot\left(I^2_{(j,i)-a} + I^2_{(j,i)-b} + I^2_{(j,i)-c} + I^2_{(j,i)-n}\right)\right) \\ &\quad \cdot d_{(i,j)} \end{aligned} \tag{3}$$

where

$d_{(i,j)}$	distance of branch (i,j);
c_{tk}	investment cost of the k conductor type, per unit of length;
c'_{tk}	cost of electrical losses in k conductor type, per unit of length and wire (phase or neutral).

Single-Phase Line Cost: The single-phase line cost of the branch (i,j) is defined by (3), where only the intensity of one phase is not zero. In this case, the intensity of the neutral is equal to the phase. Value c_{tk} is the investment cost of a single-phase type k line.

The network can be optimized with tapering conductor constraints, adding the condition

$$\begin{aligned} &t_k^{(i,j)}\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right] \\ &\qquad \geq \min_{\substack{x=adj(i)\\ x\neq j}} \left\{t_k^{(x,i)}\left[I_{(i,x)-a}, I_{(i,x)-b}, I_{(i,x)-c}\right]\right\}. \end{aligned}$$

The conductor type selected must be associated with the cost value, because they will be used in the calculus for the values of the next subtrees.

The optimization process starts in leaf nodes. When the recursive process is ended, each subtree will have associated a three-dimensional (3-D) matrix [(1)]. Each subtree can be fed with different combinations of currents in each phase. The different combinations can be represented with a matrix of *costs/intensities of phase* that have been noted: DI3-matrix. The DI3-matrix has three dimensions (one per phase) and the size is according to the level in the tree and the number of conductor types used.

The process can be simplified when customers have identical power factors. In this case, it is possible to obtain the value of the current through one phase, with the currents of the other two phases and the total power of the subtree $P_{A(j,i)}$ (4)

$$\begin{aligned} P_{A_{(j,i)}} &= \left(I_{(j,i)-a} + I_{(j,i)-b} + I_{(j,i)-c}\right)\cdot U_p \cdot \cos(\varphi) \\ \Rightarrow I_{(j,i)-c} &= \left(\frac{P_{A_{(i,j)}}}{U_p\cdot\cos(\varphi)}\right) - I_{(j,i)-a} - I_{(j,i)-b} \\ I_{(j,i)-c} &= \text{constant} - I_{(j,i)-a} - I_{(j,i)-b} \end{aligned} \tag{4}$$

where

$P_{A(i,j)}$	ipower of the customers belonging to subtree $\mathrm{A}_{(i,j)}$;
U_p	phase-neutral voltage;
$cos(\varphi)$	power factor of customers.

The result is that the DI3-matrix can be replaced by a DI2-matrix with only two dimensions. The nonzero elements of the DI3-matrix (possible values of currents) of subtree $A_{(i,j)}$ belong to a plane, and this is defined by the elements ($I_{(j,i)-a}$, $I_{(j,i)-b}$, and $I_{(j,i)-c}$). The elements of the DI2-matrix are the projection of the plane of nonzero elements in the DI3-matrix (Fig. 1).

The values of the DI2-matrix are equivalent to the values of the DI3-matrix

$$D_{(i,j)}\left[I_{(j,i)-a}, I_{(j,i)-b}\right] \triangleq D_{(i,j)}\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right].$$

The first DI2-matrices are for the leaf nodes, f. When the leaf node customer is single-phase, the matrix has 3 nonzero elements, corresponding to the current I_f of the customer through each one of the phases. If the customer is balanced three-phase, the matrix has only one element, corresponding to the balanced current through the three phases.

The costs of a subtree can be represented by a 3-D graphic (Fig. 2), of coordinates $(I_a, I_b, cost)$. The extreme points in Fig. 2 represents the cost of single-phase lines (only phase a,

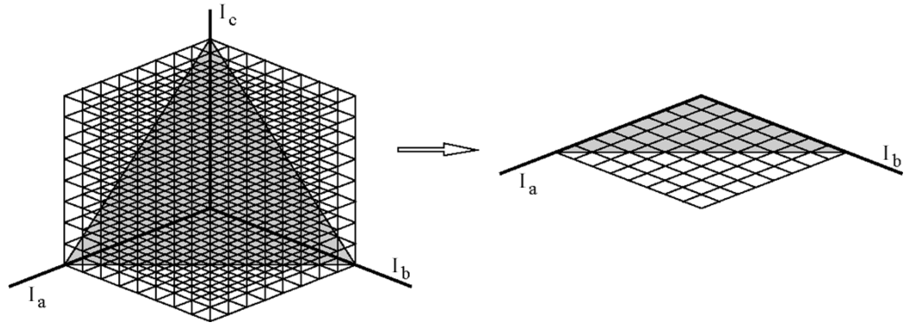

Fig. 1. DI3-matrix and equivalent DI2-matrix.

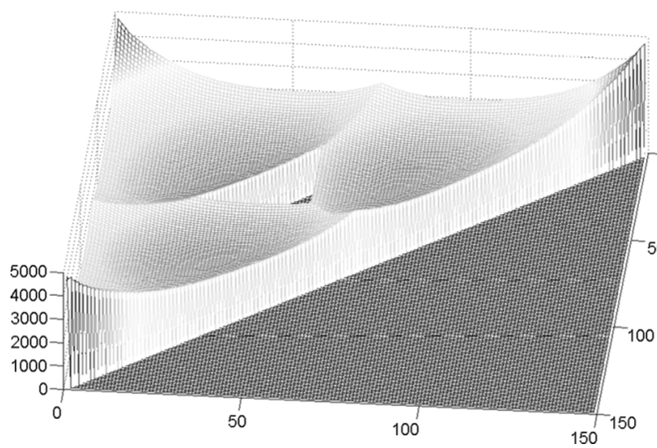

Fig. 2. Cost surface.

b or c and neutral conductors). Some values of currents by the three phases (I_a, I_b, I_c) can exist, depending of the size of the customers, so the example of Fig. 2 has not cost value by the cases near to balanced currents (center of the surface).

B. Subtree Voltage Drop

Simultaneously with the calculus of DI2-matrix, it is possible to calculate the phase-neutral voltage drops for each phase. The approximate phase-neutral voltage drops of a subtree $A_{(i,j)}$, can be calculated with (5).

Fig. 3 represents the phase-neutral voltage drops for phases a, b and c, equivalent to the example in Fig. 2, with coordinates $(I_a, I_b, \Delta U)$. Fig. 4 represents the maximal value of graphics in Fig. 3. The maximal value of the three phase-neutral voltage drops must be lower than the voltage drop limit. The ΔU-phase values decrease whit the current, but can rise when the conductor type is changed (see steps in Fig. 3)

$$\begin{aligned}
\Delta U_{A_{(i,j)}-a} &= \operatorname{mod}\left\{R_{(i,j)} \cdot I_{(j,i)-a} + R_{(i,j)} \cdot \mathcal{I}_{(j,i)-n}\right\} \\
&\quad + \max_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-a}\right\} \\
\Delta U_{A_{(i,j)}-b} &= \operatorname{mod}\left\{R_{(i,j)} \cdot I_{(j,i)-b} + \mathsf{a} \cdot R_{(i,j)} \cdot \mathcal{I}_{(j,i)-n}\right\} \\
&\quad + \max_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-b}\right\} \\
\Delta U_{A_{(i,j)}-c} &= \operatorname{mod}\left\{R_{(i,j)} \cdot I_{(j,i)-c} + \mathsf{a}^2 \cdot R_{(i,j)} \cdot \mathcal{I}_{(j,i)-n}\right\} \\
&\quad + \max_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-c}\right\}. \qquad (5)
\end{aligned}$$

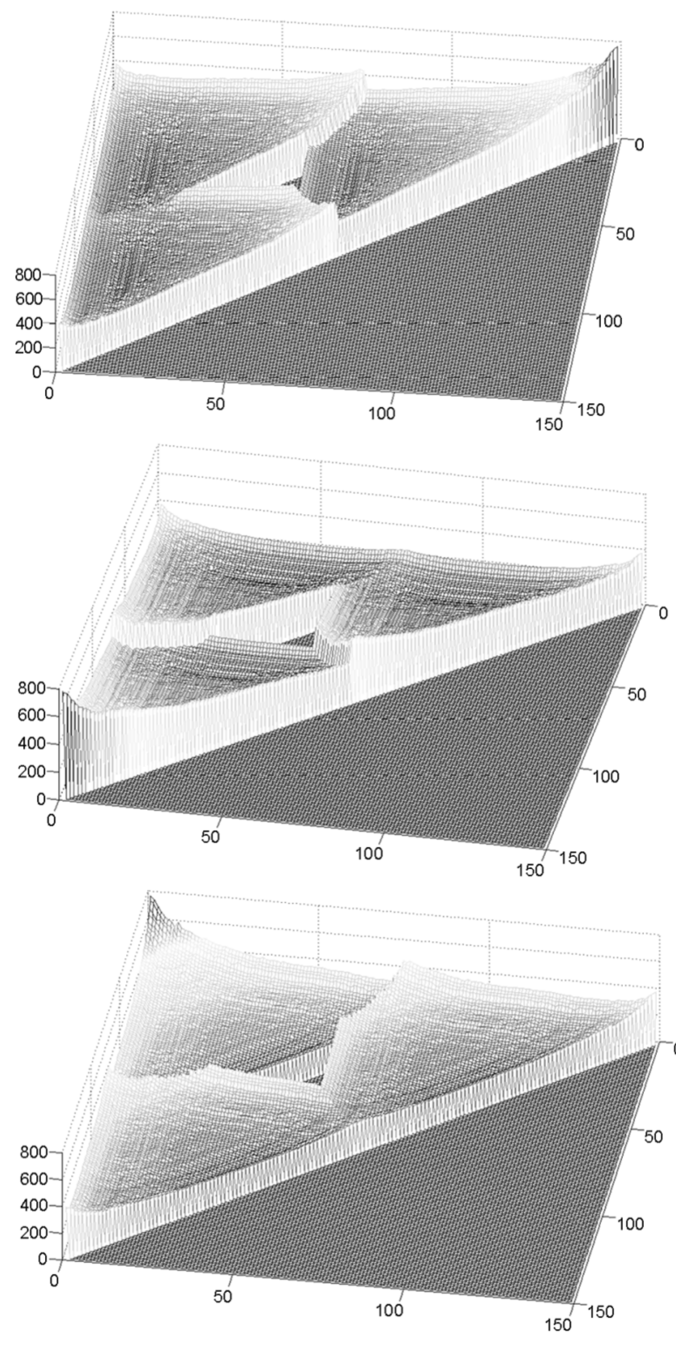

Fig. 3. Surfaces of voltage drops in phases a, b, and c.

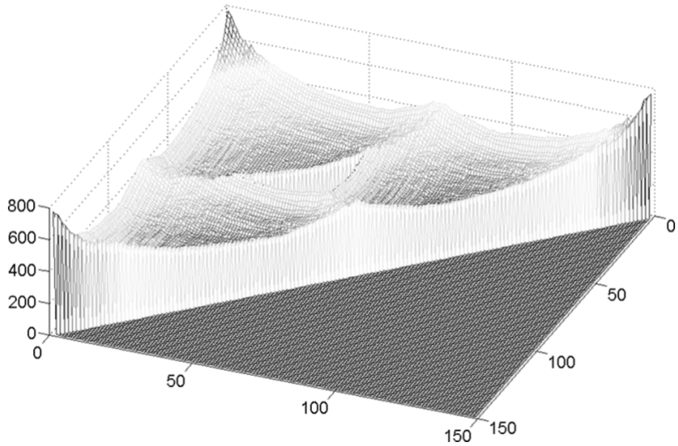

Fig. 4. Surface of maximal voltage drops.

C. Tree Cost Without Voltage Drop Constraints

Two possible cases can be considered. The first case is when the source location (substation, connection point, and so on) is fixed at node j. In this case, the optimal solution corresponds to the optimal solution of all subtrees defined by nodes adjacent to j. The solution is defined by the minimum value $D^*_{(i,j)}$ of the DI2-matrix, where

$$D^*_{(i,j)} = \min_{\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right]} \left\{ D_{(i,j)}\left[I_{(j,i)-a}, I_{(j,i)-b}\right]\right\}.$$

The second case is when the location of the source must be optimized. The following algorithm defines the operation process:

i) To calculate DI2-matrix for all the subtrees $A_{(i,j)}$.

ii) To find the minimum cost value $D^*_{(i,j)}$ in each DI2-matrix

$$D^*_{(i,j)} = \min_{\left[I_{(j,i)-a}, I_{(j,i)-b}, I_{(j,i)-c}\right]} \left\{ D_{(i,j)}\left[I_{(j,i)-a}, I_{(j,i)-b}\right]\right\}.$$

iii) To seek node k, with

$$C_{\min} = \min_k \left\{ \sum_{i=adj(k)} D^*_{(i,k)} \right\}.$$

iv) Node k with C_{min} is the *optimal position for the mv/lv substation* without voltage drop constraints.

v) The values of $D^*_{(i,k)}$ define the optimal conductors and distributions of customers to phases for each subtree $A_{(i,k)}$.

D. Tree Cost With Voltage Drop Constraints

The values of the maximal voltage drops are obtained simultaneously with the costs (step B). The optimal solution without voltage drop constraints can have a maximal value of voltage drop lower or greater than voltage drop limit. In the first case, the solution is the global optimal. In the second case, a new solution must be sought.

An approximation to the optimal solution is to obtain new optimal conductor types, with the currents obtained in step C, in such a way that the voltage drops are lower than the limit. The method proposed here is an improvement on [5], but with a 3-D matrix, corresponding to voltage drops in each phase. The method [5] considers balanced networks. If the currents through the three phases are identical, the current through the neutral conductor is zero, and the voltage drops in the three phases are the same. The network solution obtained in step C, is an unbalanced network, with different currents through each phase and with current through the neutral conductor. As the currents through each phase are different, the voltage drops are also different, and a 3-D voltage matrix must be employed by obtaining DV3-tables for cost as a function of $\Delta \mathrm{U_a}$, $\Delta \mathrm{U_b}$, and $\Delta \mathrm{U_c}$.

Therefore mathematically, the cost function of subtree $A_{(i,j)}$, with the current phase values known can be denoted by $E_{(i,j)}[\Delta U_{A(i,j)-a}, \Delta U_{A(i,j)-b}, \Delta U_{A(i,j)-c}]$, where $\Delta U_{A(i,j)-a}$, $\Delta U_{A(i,j)-b}$, and $\Delta U_{A(i,j)-c}$ are the voltage drop in each phase of subtree $A_{(i,j)}$ including branch (i,j). Recursively, the expression is (6)

$$\begin{aligned} E_{(i,j)}\left[\Delta U_{A_{(i,j)}-a}, \Delta U_{A_{(i,j)}-b}, \Delta U_{A_{(i,j)}-c}\right] &= \min_{t_k \in Y} \left\{C_{(i,j),t_k}\right\} \\ &+ \sum_{\substack{x=adj(i)\\ x\neq j}} \left(E_{(x,i)}\left[\Delta U_{A_{(x,i)}-a}, \Delta U_{A_{(x,i)}-b}, \Delta U_{A_{(x,i)}-c}\right]\right) \Big/ \\ &\Big/ \begin{cases} \Delta U_{A_{(i,j)}-a} = \Delta U_{(j,i)-a,t_k} + \max\limits_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-a}\right\} \\ \Delta U_{A_{(i,j)}-b} = \Delta U_{(j,i)-b,t_k} + \max\limits_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-b}\right\} \\ \Delta U_{A_{(i,j)}-c} = \Delta U_{(j,i)-c,t_k} + \max\limits_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-c}\right\} \end{cases} \end{aligned} \quad (6)$$

where $C_{(i,j),tk}$ is the cost with the t_k conductor type, defined by (2), for the known currents of branch (i,j) and $\Delta U_{(j,i)-a,t_k}$, $\Delta U_{(j,i)-b,t_k}$, and $\Delta U_{(j,i)-c,t_k}$ are the phase-neutral voltage drops in branch (i,j) with k type, in phases a, b, and c.

The process starts in leaf nodes and finishes in a mv/lv substation node. When the recursive process is ended, each node has one matrix: DV3-matrix associated. The DV3-matrix is defined by costs and voltage drops. These DV3-matrices have three dimensions (one per phase) and the size is according to level in the tree and number of conductor types used.

Let s *be* the mv/lv substation node. The lower value of the costs of the DV3-matrix, with the three voltage drop values lower than the voltage drop limit, define the optimal conductor types for all the branches of each subtree $A_{(k,s)}$.

E. Optimal Tree Cost With Voltage Drop Constraints

The solution obtained in step D, is an upper limit of the optimal solution. All the solutions with lower values than this, obtained without voltage drop constraints, must be analyzed. If any of them has a voltage drop below the voltage limit, then it is a better solution and becomes the new upper cost limit. When there is no solution with voltage drop below the voltage limit, they will be evaluated with the method presented in D. If the cost with voltage drop constraints is lower than the cost limit, this is the new limit. The process is applied to the adjacent nodes, while lower values are obtained. The minimum value is the optimal solution.

F. Customers With Different Voltage Drop Constraints

Industrial installations are different from distribution networks: "The maximal voltage drop constraints by the customers are different." In Spain, for example, the maximal voltage drop by lighting customers is $\pm 3\%$ (6%), and by other customers is $\pm 5\%$ (10%). The voltage drops are evaluated from leaves toward the root. The customers with low voltage drop limit must be calculated with an initial voltage drop equal to the

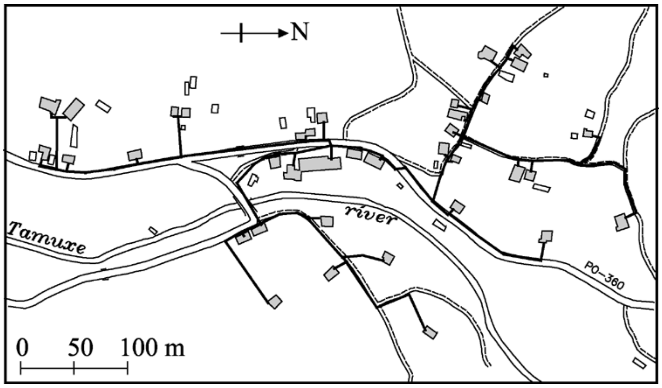

Fig. 5. Tree of lv network trajectories.

TABLE I
COSTS FOR TYPICAL SPANISH COMMERCIAL lv LINES

	c (€/m) (single-phase)	c (€/m) (three-phase)	c' (€/A^2.m) per conductor
RZ-25	4.327	7.212	0.0023
RZ-50	-	8.715	0.0011
RZ-95	-	10.818	0.0006

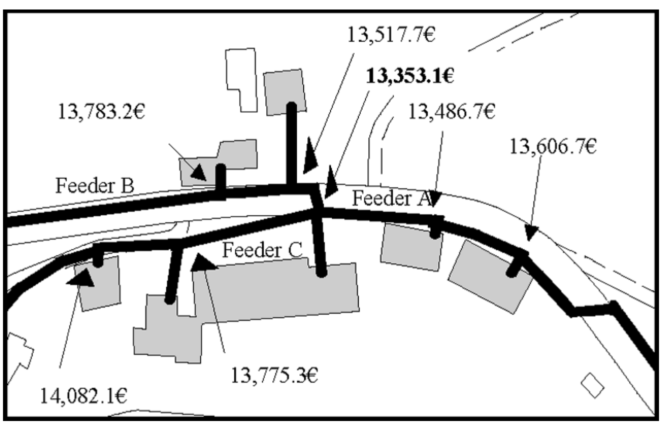

Fig. 6. Optimal mv/lv substation location and lv network costs for other substation locations.

difference between the maximal voltage drop limit and their voltage drop limit. In the Spanish case, lighting customers will be considered to have 4% of initial value.

III. APLICATION TO SINGLE-WIRE EARTH RETURN (SWER) SYSTEMS

The proposed algorithm also can be employed to solve SWER systems. In this case, the voltage drop of the three phases do not include neutral voltage drop, and (5) must be replaced by (7), as follows:

$$\begin{aligned}\Delta U_{A_{(i,j)}-a} &= \mathrm{mod}\left\{R_{(i,j)} \cdot I_{(j,i)-a}\right\} \\ &\quad + \max_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-a}\right\} \\ \Delta U_{A_{(i,j)}-b} &= \mathrm{mod}\left\{R_{(i,j)} \cdot I_{(j,i)-b}\right\} \\ &\quad + \max_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-b}\right\} \\ \Delta U_{A_{(i,j)}-c} &= \mathrm{mod}\left\{R_{(i,j)} \cdot I_{(j,i)-c}\right\} \\ &\quad + \max_{\substack{x=adj(i)\\ x\neq j}} \left\{\Delta U_{A_{(x,i)}-c}\right\}. \end{aligned} \tag{7}$$

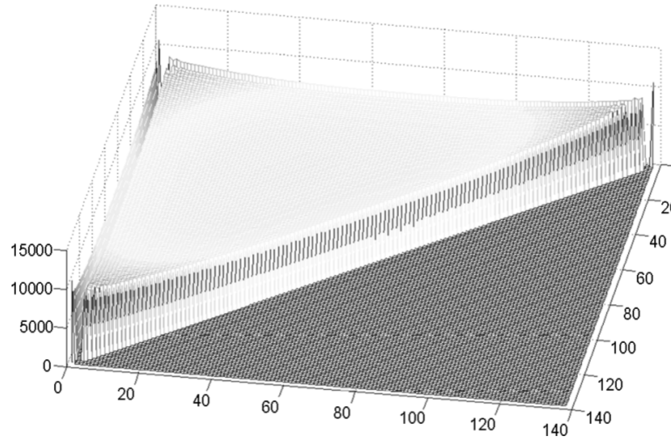

Fig. 7. Surface of costs for feeder A, without voltage drop constraints.

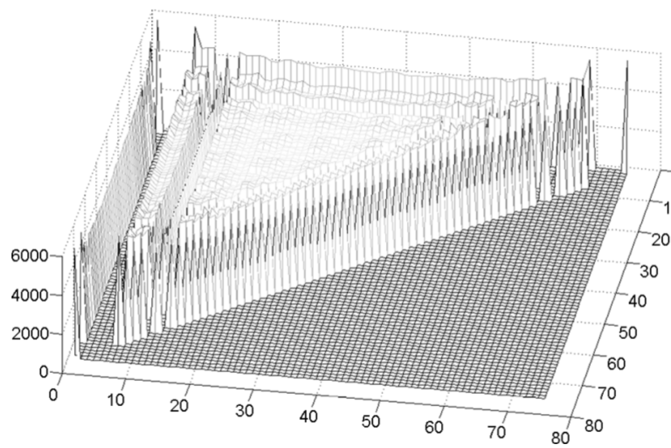

Fig. 8. Surface of costs for feeder B, without voltage drop constraints.

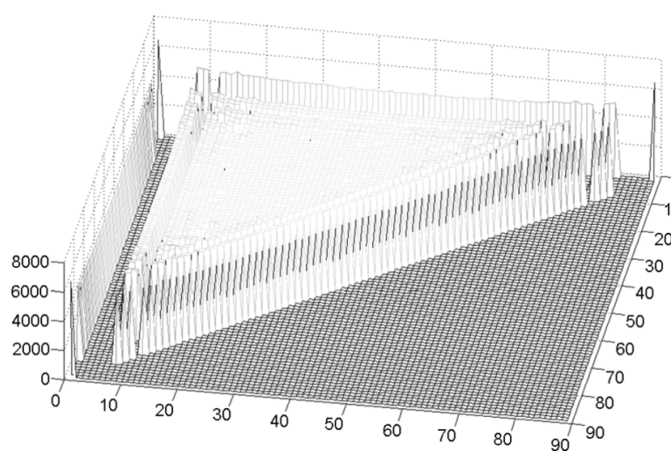

Fig. 9. Surface of costs for feeder C, without voltage drop constraints.

Each customer has associated a initial voltage drop, ΔU_i, calculated with the earth resistance and intensity of the customer I_i, and only must be considered by the phase that is assigned

$$\Delta U_i = R_{i_{earth}} \cdot I_i.$$

The limit voltage drop in each distribution of currents $I_{(j,i)-a}$, $I_{(j,i)-b}$, $I_{(j,i)-c}$ is different, because the current by the earth resistance of the hv/mv transformer (source) must be considered.

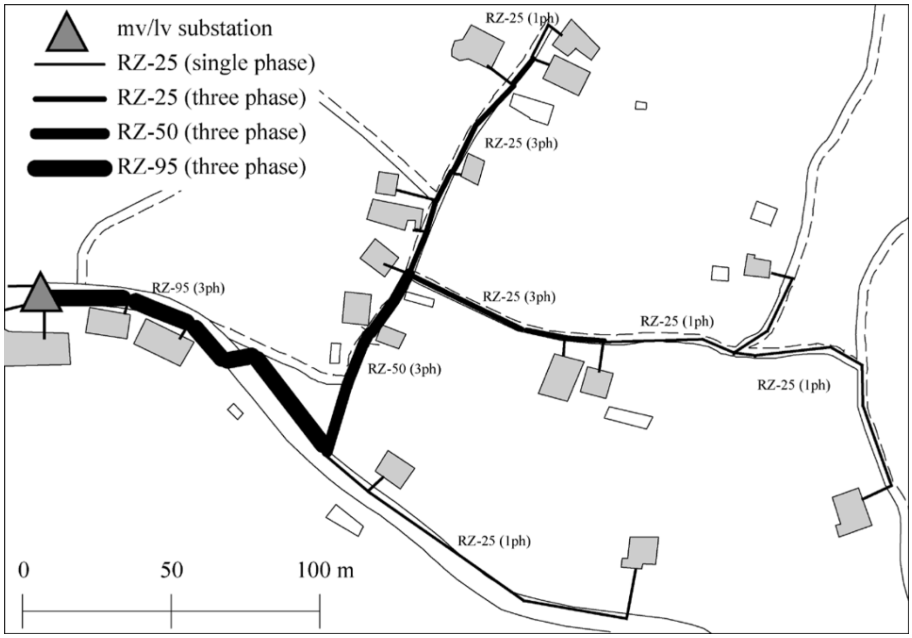

Fig. 10. Optimal subnetwork of feeder A without voltage drop constraints.

IV. Improvement of Optimization Process

To improve the optimization process two actions can be considered. The first is associated with DI2-matrix reduction, and the second, with DV3-matrix reduction.

A DI2-matrix can be reduced by the discretization of currents through each phase. So, for N intervals of I_{max}, the DI2-matrix has N elements in both dimensions.

A DV3-matrix can be reduced by the discretization of voltage-drop i. Thus, for M intervals of Δu_{max} the DV3-matrix has M elements in each dimension.

V. Results

The example represents an area of 230 000 m^2, where there are 36 customers with a total power of 27.2 kW. All the customers will be fed from an mv/lv substation, and the possible trajectories of the lv network are defined (Fig. 5).

The objectives of the algorithm are:

a) to obtain the optimal location of the mv/lv substation;
b) to assign each customer to a phase;
c) to select the conductor type for each branch.

The economic parameters employed are: 0.04 Euro/kW-losses, 25-year planning, 5% overload factor, 1% annual inflation, and 5% annual interest. The annual loss load factor employed was [20]: $\mathrm{lsf} = 0.16 * \mathrm{lf} + 0.84 * \mathrm{lf}^2$, with the load factor $\mathrm{lf} = 0.25$. Table I shows the costs for typical Spanish commercial lines, both single-phase and three-phase, by rural lv distribution.

The DI2-matrices are obtained for all the subtrees, and the sum of the adjacent subtrees is evaluated for each node. The optimal position of an mv/lv substation without voltage drop constraints is shown in Fig. 6. The cost of this node is 13 353.1 Euros.

This network has three subtrees or feeders, and the surfaces for cost in their DI2-matrix are shown in Fig. 7, 8, and 9. The

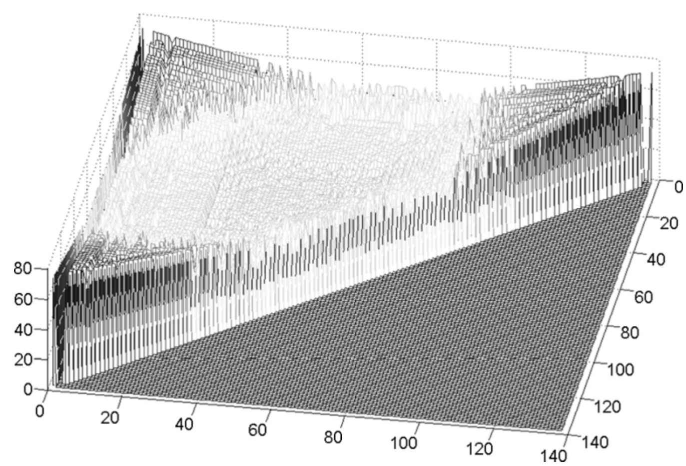

Fig. 11. Surface of maximal voltage drop for feeder A with configuration 1.

minimum cost of each surface are: feeder A with 5,838.5 Euros and currents (44 A, 45 A, 48 A), feeder B with 3,511.3 Euros and currents (26 A, 23 A, 25 A), and feeder C with 4,003.3 Euros and currents (27 A, 28 A, 33 A).

The total cost of the lv network without voltage drop constraints is 13 353.1 Euros and currents (100 A, 99 A, 100 A).

The conductor types of feeder A are shown in Fig. 10 (configuration 1).

If a maximal voltage drop constraint of $\pm 5\%$ is considered (10% total), feeders A and B must be recalculated.

Feeder A was obtained without voltage drop constraints, and the values of the phase voltage drops are (12.7%, 5.46%, 9.54%). Fig. 11 shows the maximal voltage drop for the feeder A solutions. A new configuration of feeder A is obtained with the same assignation of customers to phases as the first configuration, but with different conductor types, and the algorithm proposed in Section II, step D is applied. This second configuration has a cost of 5950.4 Euros and the conductor types selected are shown in

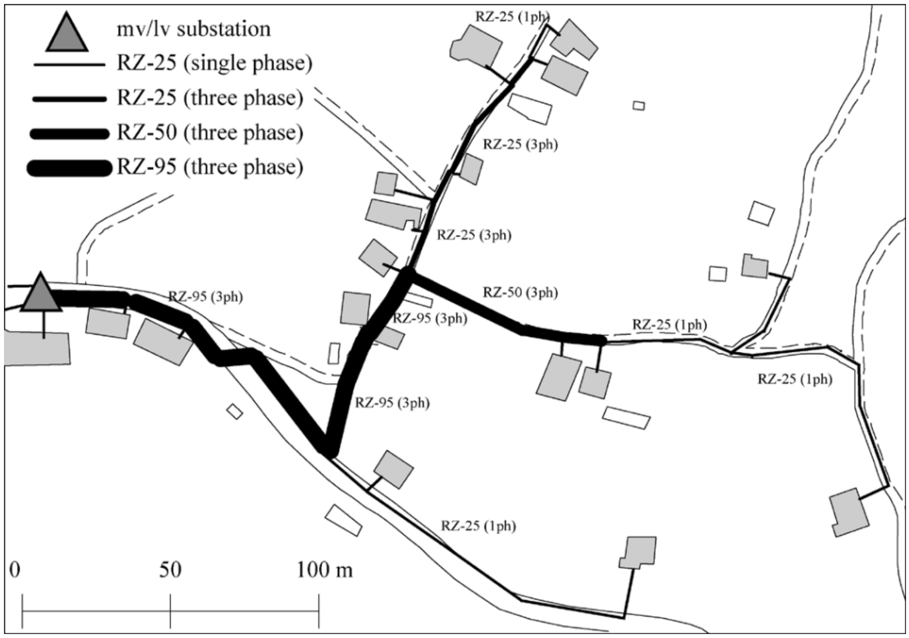

Fig. 12. Subnetwork of feeder A with voltage drop constraints and the same assignation of customers to phases as Fig. 11.

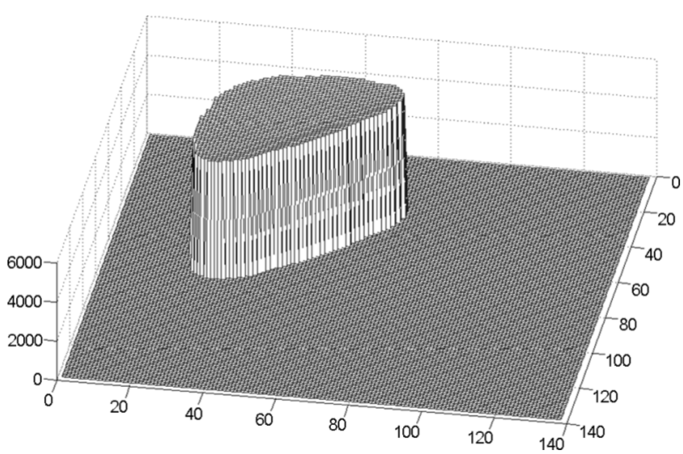

Fig. 13. Fragment of the surface of costs for feeder A (Fig. 7), without voltage drop constraints and cost lower than 5950.4 Euros.

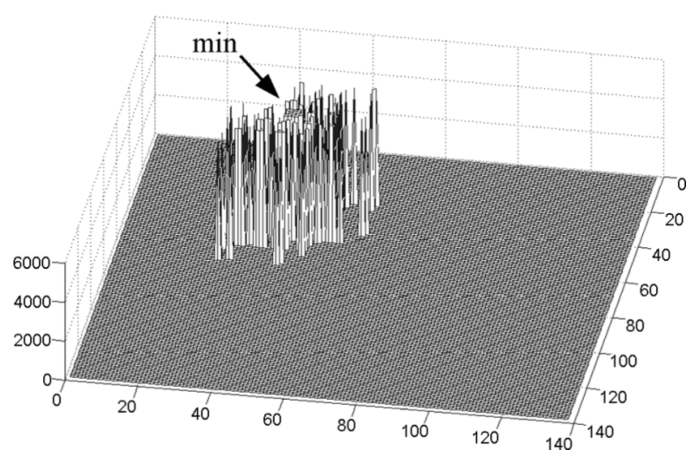

Fig. 14. Surface of costs for feeder A, with voltage drop constraints and cost lower than 5950.4 Euros.

Fig. 12. The cost of the second configuration is an upper limit for searching for better configurations. The new solutions are sought using the set of solutions without voltage drop constraints (Fig. 7) with a lower cost than 5950.4 Euros (Fig. 13).

These cases are evaluated with voltage drops, and the new lower costs are considered as new limits and provisional optimal solutions (Fig. 14).

The best solution with voltage drop constraints for feeder A has a cost of 5901.9 Euros, with currents of (28 A, 50 A, 59 A) and voltage drop of (9.95%, 9.95%, 6.73%) (Fig. 15).

The process must be replayed by feeders B and C. The optimal configuration of feeder B without voltage drop constraints is 3511.3 Euros and has a voltage drop above the voltage drop limit (7.61%,8.96%,14.33%). The solution with minimal cost and maximal voltage drop below the limit is 3625.6 Euros. This value is an upper limit in the search for other solutions with voltage drop constraints. The second configuration of feeder B evaluated with constraints (Section II, step D) is 3,606.1 Euros. The global optimal of feeder B is 3,593.2 Euros and the currents of the feeder are (24 A, 24 A, 26 A).

The initial solution of feeder C has voltage drop below of the limit, and this is the optimal solution.

The total cost of the network with voltage drop constraints is 13 498.4 Euros $(5901.9\,\mathrm{Euros}+3593.2\,\mathrm{Euros}+4003.3\,\mathrm{Euros})$ and the network is shown in Fig. 16.

The optimal solution, considering all customers to be balanced three-phase with the same power, has a greater cost than this case. The cost of the balanced network is 15 059.1 Euros, that is 11.6% greater than the unbalanced network with a cost of 13 498.4 Euros. The principal difference is the investment cost, because it is not possible to employ single-phase lines. The optimal balanced network fulfils voltage drop constraints. The optimal balanced network is employed with unbalanced customers, and power losses and their cost are more than those calculated. Furthermore, the voltage drop with the unbalanced customers is more than that calculated with balanced customer.

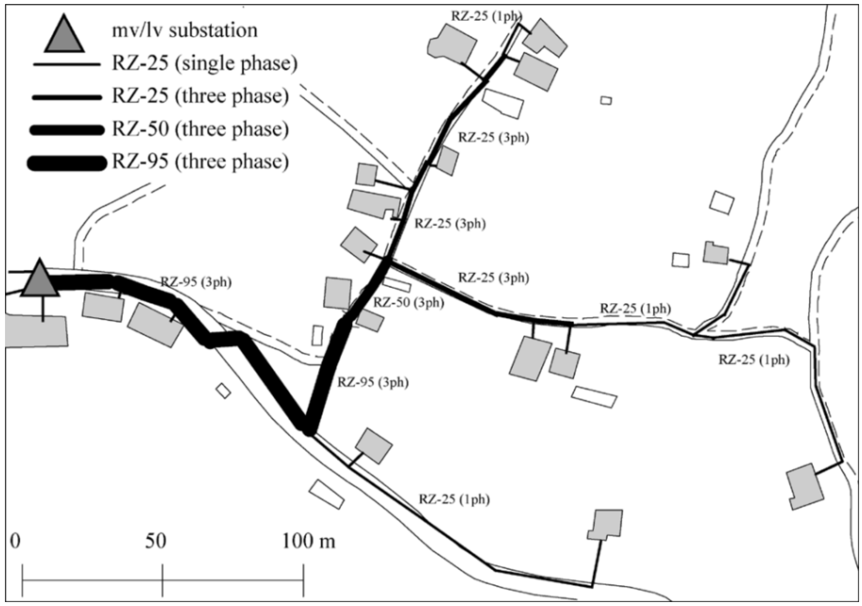

Fig. 15. Optimal subnetwork of feeder A with voltage drop constraints.

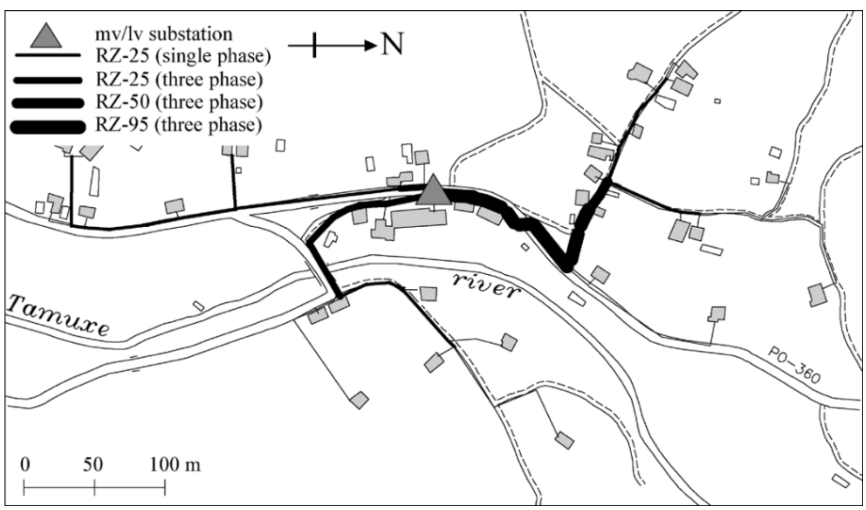

Fig. 16. Optimal unbalanced network with voltage drop constraints.

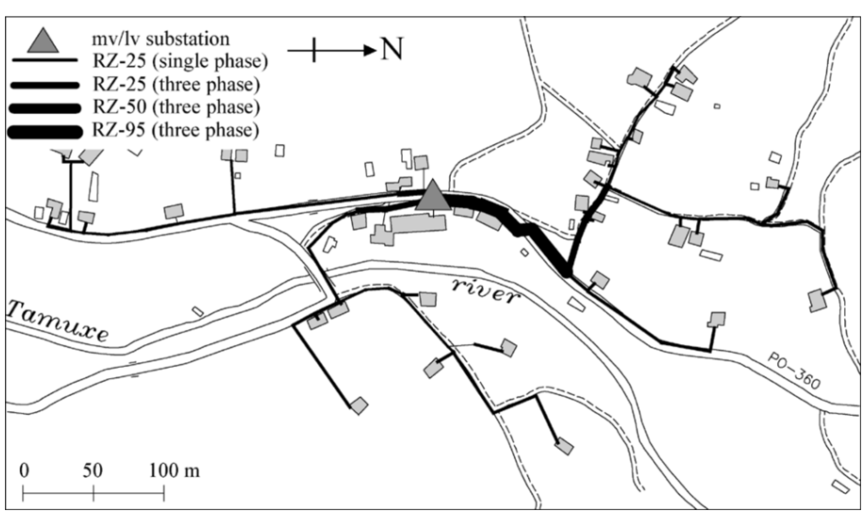

Fig. 17. Optimal balanced network with voltage drop constraints.

Feeder A in the example has a voltage drop of 6.3% with balanced customers, but for the same network with unbalanced customers this is higher. Fig. 17 represents the balanced network. The section of some lines are lower than for the unbalanced network, but all the lines are made up of three-phase conductors.

VI. Conclusions

The proposed method optimizes distribution and industrial networks using a dynamic programming optimization algorithm. This method considered the optimal assignation of single-phase customers to phases, the unbalanced currents, power losses and voltage drops, the possibility of employing single-phase and three-phase conductors and multiple conductor types.

The algorithm can optimize the source location (substation), the conductor types and the assignation of customers to phases, with voltage drop and capacity constraints. The method presented optimizes lv networks, but it can be used to optimize mv networks with single-phase customers and industrial installations and by SWER systems.

The saving obtained for the unbalanced networks optimized with this algorithm is greater than 10%, with regard to those optimized with other algorithms that consider balanced networks.

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