Synchronization of Asynchronous Wind Turbines

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***Abstract*—In this paper, a theoretical analysis and explanation of synchronization phenomena in wind parks with asynchronous generators is presented.**

***Index Terms*—Asynchronous wind turbine, slow voltage fluctuation, synchronization of induction machines, wind energy, wind park.**

I. Introduction

IN STEADY-STATE, the synchronization of wind turbines in a wind park occurs when the blades of two different wind turbines have the same rotational speed. In this case, the relative angle between their blades is constant. In this situation, the steady-state fluctuations of the electrical and mechanical variables of wind turbines have the same values and they can produce significant effects on the electrical power system, because they occur simultaneously.

Synchronization of wind turbines does not mean they operate at network synchronous speed. It means that several wind turbines operate at exactly the same speed. In fact, synchronization can occur when two fixed-speed asynchronous wind turbines generate real power, so they are operating over synchronous speed.

The power injected into the electrical network from these machines oscillates due to several fluctuation phenomena, such as wind shear effect, rotational sampling and tower shadow, with an amplitude of about 20% of the mean power value, according to some authors and measurements [1]–[3]. These fluctuations imply variations in currents injected into the network that finally become voltage variations with the same frequency. The final effect can be the existence of electrical fluctuations, such as voltage flicker, which eventually results in a negative influence on power quality. This effect also depends on parameters of the network at the point of common connection, e.g., its short-circuit power and the inductive to resistive relationship of this short-circuit feature.

Generally, the fact is assumed that the effect of a wind park on the electrical network can be calculated according to the square root of n rule [1], [4], [5], where n is the number of wind turbines in the wind park, assuming they are of the same kind. According to this rule, the flicker index P_{st} of the whole wind park is estimated as the square root of n times the index due to a single machine. Some authors have taken measurements which agree with this idea [4], but others [1] hold that they have observed a trend to synchronization in groups of wind turbines. If this really happens, the total effect should be higher and, consequently, should be quantified differently. So, there might be reasonable doubts as to whether or not the square root of n rule holds, which makes the discussion interesting. Conclusions about the ability of several wind turbines in a wind park to synchronize can help in deciding if the mentioned rule is applicable.

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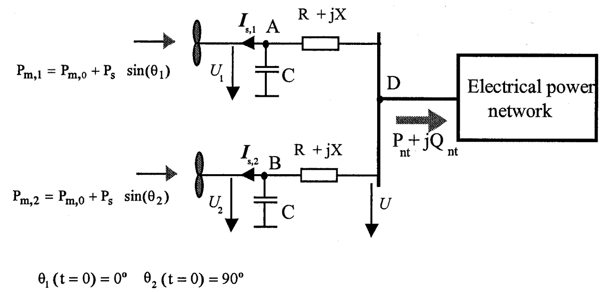


Fig. 1. Simulation of two asynchronous wind turbines.

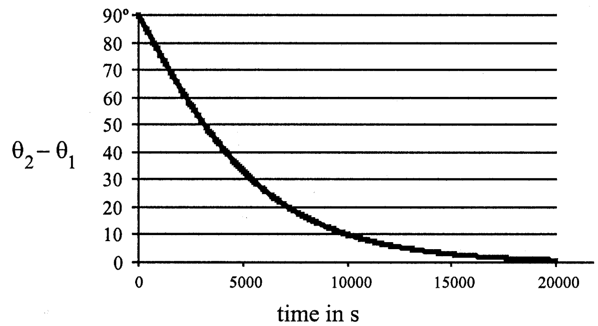
By simulation it is known that two identical wind turbines, as shown in the scheme of Fig. 1, with the same operating point conditions and connected to a common bus bar, tend to synchronize when mechanical power fluctuations exist [6], although their initial angles are different.

Synchronization, as shown in Fig. 2(a) and (b) and Fig.3, makes the difference between mechanical angles tend to zero and mechanical and electrical powers to be equal. The synchronization process, as can be seen in the same figures, has a very slow time constant.

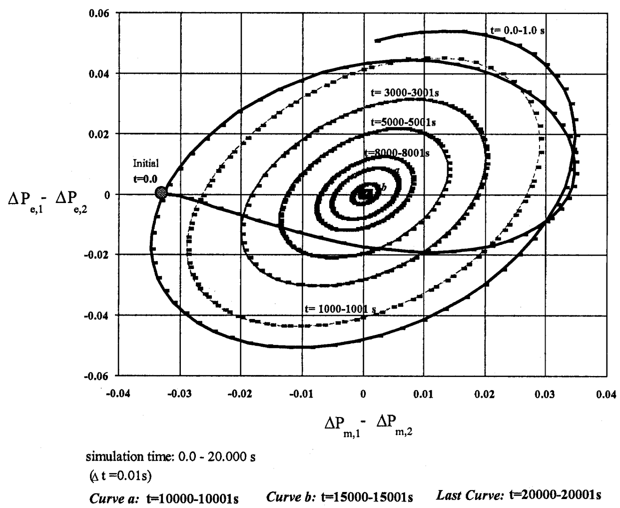
Some explanations have been given to the process of asynchronous wind turbines synchronization in a wind park assuming steady-state models [8].

The purpose of this paper is to look for an explanation for synchronization conditions of asynchronous wind turbines connected to the same point, by using a linear dynamic model derived from the well-known third-order dynamic model widely described in [9]. This model was already used in [10] for deriving a steady-state model for wind turbines with mechanical fluctuations.

The linear dynamic model [11] is used with the application of mechanical sinusoidal fluctuations ΔP_m. Mechanical sinusoidal fluctuations are fluctuations with a sinusoidal shape, superimposed to the constant mechanical power. This model for mechanical power fluctuations tries to simulate the above-mentioned real effects: tower shadow, wind shear, and rotational sampling. Measurements taken by the authors in a wind turbine [11] show that this can be a correct way of simulating mechanical power. In [11], a spectral analysis of real power generated by a wind turbine is shown, where fluctuations with a frequency



(a)



(b)

Fig. 2. (a) Evolution of mechanical angles difference between two identical wind turbines over time. (b) Evolutions of electrical and mechanical powers for the synchronization process.

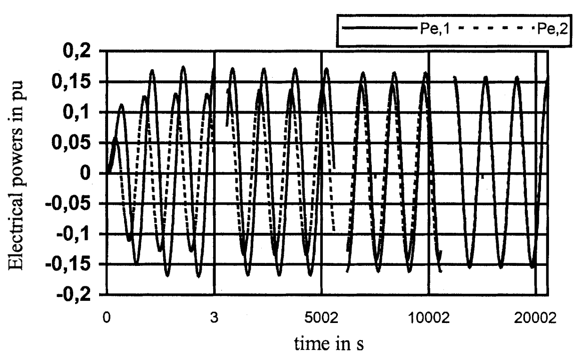


Fig. 3. Evolution of electrical powers.

of $3P$ are observed to be dominant. Other authors have observed the same effect [1] and [2], although some others have also reported fluctuations of $1P$ and $3nP$ $(n \geq 1)$ [4].

The results of simulation with the linear model are used to calculate variations in constant values of electrical and mechanical powers δP_m, δP_e. These powers are applied to dynamic models and modifications of rotational and mechanical angles are obtained that can finally lead to synchronization of wind turbines, depending on wind turbine parameters and conditions.

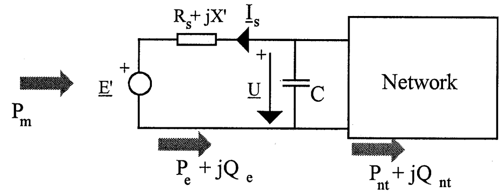


Fig. 4. Scheme of an asynchronous generator in front of a network.

II. Induction Wind Turbine Models

The induction generator can be modeled as a Thevenin equivalent voltage source $\underline{E}'$ behind the impedance $R + jX'$. A scheme of such an equivalent in front of the electrical network is shown in Fig. 4 [9]. This dynamic model is defined by considering the machine operating in a balanced way and neglecting the electromagnetic transients of the stator, and it is known as the third-order induction machine model.

The value of $\underline{E}'$ can be calculated by integrating the following complex equation:

$$\frac{d\underline{E}'}{dt} = -j\omega_s s \underline{E}' - \frac{1}{T_0'}\left(\underline{E}' - j(X_0 - X')\underline{I}_s\right) \quad (1)$$

where the constants T_0', X' and X_0 can be calculated as follows:

$$T_0' = \frac{X_r + X_m}{2\pi f_s R_R} \quad X_0 = X_r + X_m \quad X' = X_s + \frac{X_r X_m}{X_r + X_m}$$

where

- X_r rotor leakage reactance;
- X_s stator leakage reactance;
- X_m magnetizing reactance;
- R_s stator resistance;
- R_r rotor resistance;
- f_s network frequency and $\omega_s = 2\pi f_s$;
- I_s stator current.

The electrical equation of the stator current is

$$\underline{I}_s = \frac{\underline{U} - \underline{E}'}{R_s + jX'} \quad (2)$$

where $\underline{U}$ is the stator voltage.

The electromechanical equation (3) is given by the balance of electrical and mechanical torques

$$-\frac{P_m}{1-s} + P_e' = -2H\frac{ds}{dt} \Rightarrow P_m - P_e = 2H(1-s)\frac{ds}{dt} \quad (3)$$

where

- P_m mechanical power;
- H inertia constant, obtained from the moment of inertia;
- $P_e = P_e'(1-s)$ real power injected into the network and P_e.

The mechanical power can be expressed by the following terms:

$$P_m(t) = P_{m,0} + \Delta P_m(t) \quad (4)$$

where

$P_{m,0}$ 10-min mean value of the power of the wind turbine, obtainable as a function of the wind speed, which follows a Weibull or a Rayleigh distribution [12], [13];

ΔP_m power caused by tower shadow, wind shear, and rotational sampling.

In this paper, the following hypotheses are considered.

1) The $P_{m,0}$ component of mechanical power is constant.
2) The ΔP_m mechanical power can be expressed as

$$\Delta P_m(t) = P_s \sin(\theta(t)) \tag{5}$$

where P_s is the amplitude of mechanical fluctuations; and $\theta(t)$ is the mechanical angle of the wind turbine blades, defined as

$$(1-s)\Omega_s = \frac{d\theta(t)}{dt} \Rightarrow \theta(t) = \theta_0 + \int_{t_0}^{t} (1-s)\Omega_s \, dt \tag{6}$$

where

s slip of the induction generator;

$\Omega_s = 3 \cdot 2\omega_s/(rp)$ synchronous speed in rad/s of all the fluctuating effects considering a three-bladed machine, where the synchronous speed of the turbine is $2\omega_s/(rp)$;

ω_s synchronous speed in rad/s of the induction generator;

r gear box ratio, with a value of 1 : 44.38 in the case analyzed here;

p number of poles, which is four in this case.

The electrical power P_e is calculated by

$$P_e = P'_e(1-s) = \text{Re}\left\{\underline{E}' \, \underline{I}_s^*\right\}. \tag{7}$$

If the induction generator is assumed to be in the initial steady-state situation, an operating point defined by the values E'_0, s_0, I_0, P_0, and U_0, then the value of a small variation of E', $\Delta E'$, taking (1) and (2) into account, can be calculated by integrating the following equation [7], [10]:

$$\frac{d\Delta \underline{E}'}{dt} = -\underline{z}\Delta\underline{E}' - j\omega_s \Delta s \underline{E}'_0 + \underline{z}'\Delta\underline{U} - j\omega_s\Delta s \Delta\underline{E}' \tag{8}$$

where

$$\underline{z} = j\omega_s s_0 + \frac{1}{T'_0}\left(1 + \frac{j(X_0 - X')}{R_s + jX'}\right) \quad \underline{z}' = \frac{1}{T'_0}\,\frac{j(X_0 - X')}{R_s + jX'}.$$

Also, the following can be written for the electromechanical equations (3):

$$\Delta P_m - \Delta P_e = 2H(1 - s_0 - \Delta s)\frac{d\Delta s}{dt} \tag{9}$$

$$\frac{d\Delta\theta}{dt} = -\Omega_s \Delta s. \tag{10}$$

A small change in the electrical power (7), ΔP_e, is calculated by

$$\Delta P_e = \text{Re}\left\{-\underline{y}\Delta\underline{E}'^* + \underline{I}^*_{s,0}\Delta\underline{E}' - \underline{y}'\Delta\underline{E}'\Delta\underline{E}'^* + \underline{y}'\Delta\underline{E}'\Delta\underline{U}^* + \underline{y}''\Delta\underline{U}^*\right\} \tag{11}$$

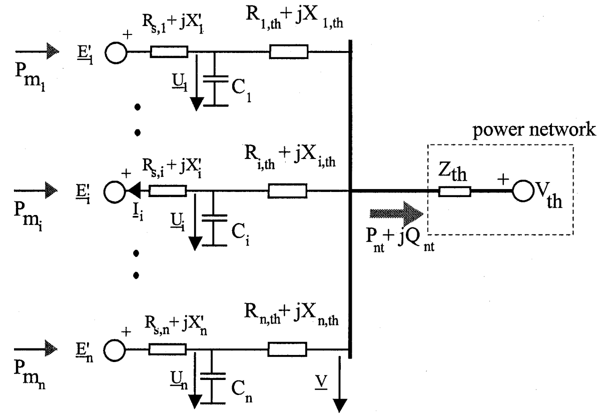


Fig. 5. Scheme of a wind park in front of an electrical network represented by its Thevenin equivalent.

where

$$\underline{y} = \frac{\underline{E}'^*_0}{R_s - jX'} \quad \underline{y}' = \frac{1}{R_s - jX'} \quad \underline{y}'' = \frac{\underline{E}'_0}{R_s - jX'}$$

and the mechanical power (5), using (6), is

$$\Delta P_m(t) = P_s \sin(\theta_0 + \Omega_0 t + \Delta\theta) \tag{12}$$

where $\Omega_0 = (1 - s_0)\Omega_s$.

For the case of a single wind turbine and assuming small changes in (8)–(11), that is to say $\Delta E'\Delta s \approx 0$, $\Delta s \ll s_0$, $\Delta E'\Delta E'^* \approx 0$, $\Delta\underline{E}'\Delta\underline{U} \approx 0$, and $\sin(\Delta\theta) = \Delta\theta \approx 0$, a linear differential first-order system results [11], [14], with a shape such as the following:

$$\dot{x} = Ax + b\Delta P'_m + c\Delta U \tag{13}$$

where x is a vector representing s and the real and imaginary components of $\underline{E}'$.

III. Wind Park Model

Applying the dynamic model to a wind park with n wind turbines, such as that schemed in Fig. 5, the following equations hold [7]:

- Network nodal analysis:

$$\Delta\underline{U} = \underline{K}\Delta\underline{E}'. \tag{14}$$

- Electrical equation (8), taking (14) into account:

$$\frac{d}{dt}\left[\Delta\underline{E}'_i\right] = \underline{B}\Delta\underline{E}' + \underline{C}\Delta s - j\omega_s\left[\Delta s_i \Delta\underline{E}'_i\right] \tag{15}$$

where the subscript i is used for the ith machine of the wind park.

Equation (15) can be split into its components

$$\frac{d}{dt}\begin{bmatrix}\Delta E'^r_i \\ \Delta E'^m_i\end{bmatrix} = B'\begin{bmatrix}\Delta E'^r_i \\ \Delta E'^m_i\end{bmatrix} + C'\Delta s + \omega_s\begin{bmatrix}\Delta s_i \Delta E'^m_i \\ -\Delta s_i\Delta E'^r_i\end{bmatrix} \tag{16}$$

where B' and C' are real matrices.

The electromechanical equations (9) and (10) can now be expressed as

$$\Delta P_{m,\,i} - \Delta P_{e,\,i} = 2H_i(1 - s_i) = h_i \frac{d\Delta s_i}{dt} \tag{17}$$

$$\frac{d\Delta\theta_i}{dt} = -\Omega_s \Delta s_i. \tag{18}$$

The electrical power $\Delta P_{e,i}$, taking (11) and (14) into account, can be calculated as

$$[\Delta P_{e,\,i}] = M\Delta E' + \text{Re}\left\{-\left[\underline{y}'_i \Delta\underline{E}'_i \Delta\underline{E}'^{\,*}_i\right] + \left[\Delta\underline{E}'_i \underline{N} \Delta\underline{E}'\right]\right\} \tag{19}$$

where M is a real matrix.

Furthermore, the mechanical power (12) is expressed as

$$\Delta P_{m,\,i} = P_{s,\,i} \sin(\theta_{0,\,i} + \Omega_{0,\,i}t + \Delta\theta_i) \tag{20}$$

where $\Omega_{0,i} = (1 - s_{0,i})\Omega_s$.

A. Mechanical Power Sinusoidal Fluctuations

Assuming small changes: $\Delta E'_i \Delta s_i \approx 0$, $\Delta E'_i \Delta E'^{*}_j \approx 0$, $\Delta\underline{E}' \Delta\underline{U} \approx 0$, and $\sin(\Delta\theta_i) = \Delta\theta_i \approx 0$, and for $(i, j = 1, \ldots, n)$, (16), (17), (19), and (20) can be expressed in a steady-state situation by a Fourier transform as

$$\underline{X} = \underline{D}\Delta\underline{P} \tag{21}$$

where

$$\underline{D} = (j\Omega 1 - A)^{-1}H$$

where

- Ω imaginary variable in Fourier analysis;
- 1 unitary matrix;
- H diagonal matrix of components $1/h_i$;
- A real matrix with the relations between the variables $\Delta E'_i$, Δs_i and $\Delta P_{e,i}$. (this matrix has not been included here, but is given in Appendix B);
- ΔP complex vector with the values of $\Delta P_{m,i}$.

Now two different cases can be considered.

In the first case, for n identical wind generators and assuming identical mean values for the mechanical powers $P_{m,1,0} = \cdots = P_{m,n,0}$ and equal values for the interconnection lines, given by $R_{1,th} + jX_{1,th} = \cdots = R_{n,th} + jX_{n,th}$, (21) has a symmetrical structure in blocks. As the relations and notation of X and ΔP are cumbersome, they are given in Appendix C. Using (21), $\Delta E'$, Δs and $\Delta\theta$ can be defined by

$$\Delta E^r_i(t) = X^r_i \sin(\Omega_0 t) + X^m_i \cos(\Omega_0 t) \tag{22.a}$$

$$\Delta E^m_i(t) = X^r_{(i+1)} \sin(\Omega_0 t) + X^m_{(i+1)} \cos(\Omega_0 t) \tag{23.a}$$

$$\Delta s_i(t) = X^r_{(i+2)} \sin(\Omega_0 t) + X^m_{(i+2)} \cos(\Omega_0 t) \tag{24.a}$$

$$\Delta\theta_i(t) = -\left(\frac{\Omega_s}{\Omega_0}\right)\left(X^m_{(i+2)} \sin(\Omega_0 t) - X^r_{(i+2)} \cos(\Omega_0 t)\right) \tag{25.a}$$

where all variables have the same rotational speed Ω_0.

There is a second case when the operating points of the wind turbines are different. In this case, (22.a)–(25.a) can be expressed in the following way:

$$\Delta E^r_i(t) = \sum_{k=1}^{n} X^{(k)^r}_i \sin(\Omega_{0,\,k}t) + \sum_{k=1}^{n} X^{(k)^m}_i \cos(\Omega_{0,\,k}t) \tag{22.b}$$

$$\Delta E^r_i(t) = \sum_{k=1}^{n} X^{(k)^r}_{i+1} \sin(\Omega_{0,\,k}t) + \sum_{k=1}^{n} X^{(k)^m}_{i+1} \cos(\Omega_{0,\,k}t) \tag{23.b}$$

$$\Delta s^r_i(t) = \sum_{k=1}^{n} X^{(k)^r}_{i+2} \sin(\Omega_{0,\,k}t) + \sum_{k=1}^{n} X^{(k)^m}_{i+2} \cos(\Omega_{0,\,k}t) \tag{24.b}$$

$$\Delta\theta_i(t) = -\left(\frac{\Omega_s}{\Omega_0}\right)\left(X^{(k)^m}_{(i+2)} \sin(\Omega_0 t) - X^{(k)^r}_{(i+2)} \cos(\Omega_0 t)\right) \tag{25.b}$$

where $\Omega_{0,k}$ are different frequencies.

If small changes are now considered, the following expression is valid:

$$\begin{bmatrix} \delta E'^r_i \\ \delta E'^m_i \end{bmatrix} \equiv \omega_s \begin{bmatrix} \Delta s_i(t)\Delta E'^m_i(t) \\ -\Delta s_i(t)\Delta E'^r_i(t) \end{bmatrix} \tag{26}$$

$$[\delta P_{e,\,i}] = M\delta E' + \text{Re}\left\{-\left[\underline{y}'_i \Delta\underline{E}'_i(t) \Delta\underline{E}'^{\,*}_i(t)\right] + \left[\Delta\underline{E}'_i(t) \sum_{k=1}^{m} \underline{y}'_k \Delta\underline{E}'_k(t)\right]\right\} \tag{27}$$

$$\begin{aligned} \delta P_{m,\,i} &= P_{s,\,i} \sin(\theta_{0,\,i} + \Omega_{0,\,i}t + \Delta\theta_i) \\ &\quad - P_{s,\,i} \sin(\theta_{0,\,i} + \Omega_{0,\,i}t) \\ &\approx P_{s,\,i} \cos(\theta_{0,\,i} + \Omega_{0,\,i}t)\Delta\theta_i. \end{aligned} \tag{28}$$

Equations (26)–(28) have constant values when $\Delta E'$, Δs, and $\Delta\theta$ have the same frequency Ω_0. For the sake of simplicity, the analytical way of obtaining (26)–(28), taking (22.a)–(25.a) into account, has been skipped and is given in Appendix D.

With $\delta P_{m,i}$ and $\delta P_{e,i}$ the following equations are obtained:

$$\delta P_{m,\,i} - \delta P_{e,\,i} - \delta P'_{e,\,i} = h_i \frac{d\delta s_i}{dt} \tag{29}$$

$$\frac{d\delta\theta}{dt} = -\Omega_s \delta s \tag{30}$$

where $\delta P'_{e,i}$ is an additional electrical power term produced by slip variations δs_i, and can be defined by the following equation:

$$\frac{d}{dt}[\delta E'] \equiv 0 = B'\delta E' + C'\delta s \tag{31}$$

$$[\delta P'_{e,\,i}] = -M(B')^{-1}C'\delta s. \tag{32}$$

Finally, a flow diagram for the simulation of a wind park can be seen in Fig. 6.

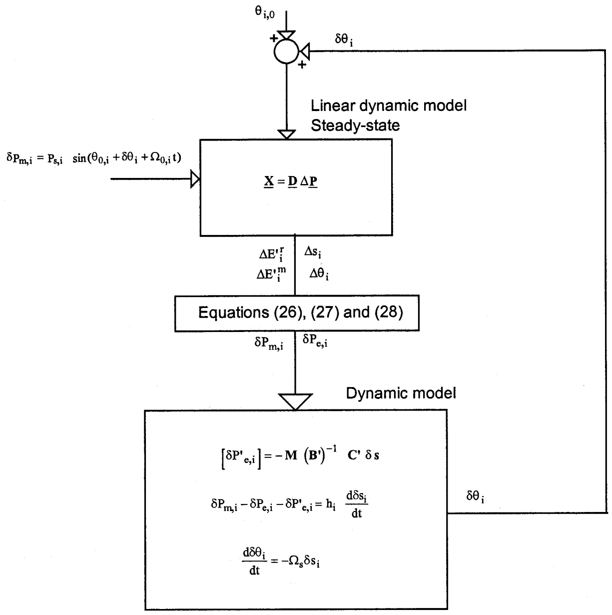


Fig. 6. Scheme of the simulation process.

IV. Analysis of the Synchronization Process

The analysis of the synchronization process is made for a wind park with identical wind turbines and mean operating point conditions. Random variations around these values do not affect it.

In the other cases, powers do not tend to equal values and, consequently, the synchronization phenomenon is not possible.

The analysis of synchronization can be defined taking one of the wind turbines as a reference. So, (29) and (30), with machine 1 as reference, are expressed as

$$(\delta P_{m,i} - \delta P_{m,1}) - (\delta P_{e,i} - \delta P_{e,1}) - (\delta P'_{e,i} - \delta P'_{e,1}) = h\,\frac{d\delta s_{i,1}}{dt} \tag{33}$$

$$\frac{d\delta\theta_{i,1}}{dt} \equiv \frac{d(\delta\theta_i - \delta\theta_1)}{dt} = -\Omega_s \delta s_{i,1} \tag{34}$$

where

$$\begin{aligned}
\delta P_{m,i} - \delta P_{m,1} &= K'^{m}_{i,1}(P^2_{s,i} - P^2_{s,1}) + K^{m}_{i,1} P_{s,i} P_{s,1} \sin(\theta_i - \theta_1)\\
K'^{m}_{i,1} &= \frac{\Omega_s}{\Omega_0} D^r_{i+2,i+2}\\
K^{m}_{i,1} &= \frac{\Omega_s}{\Omega_0} D^r_{i+2,1+2}\\
\delta P_{e,i} - \delta P_{e,1} &= K'^{e}_{i,1}(P^2_{s,i} - P^2_{s,1}) + K^{e}_{i,1} P_{s,i} P_{s,1} \sin(\theta_i - \theta_1)\\
K'^{e}_{i,1} &= M_{i+2,i}(\Lambda^{xx}_{i,i,i} - \Lambda^{xx}_{i,1,1}) + M_{i+2,i+1}(\Lambda'^{xx}_{i,i,i} - \Lambda'^{xx}_{i,1,1})\\
K^{e}_{i,1} &= M_{i+2,i}\Lambda^{xy}_{i,i,1} + M_{i+2,i+1}\Lambda'^{xy}_{i,i,1}\\
\delta P'_{e,i} - \delta P'_{e,1} &= D^e_{i,1}(\delta s_i - \delta s_1).
\end{aligned}$$

Equation (33) means that the following powers exist:

1) constant power, $K'^{m,e}_{i,j}(P^2_{s,i} - P^2_{s,j})$, produced by different mechanical power fluctuation levels;
2) synchronization power, $K^{m,e}_{i,j}\sin(\theta_i - \theta_j)$, function of power mechanical fluctuation levels and difference between mechanical angles;
3) damping power, $D^e_{i,j}(\delta s_i - \delta s_j)$. The constant ($K'^{m,e}_{i,j}$), synchronization ($K^{m,e}_{i,j}$), and damping ($D^e_{i,j}$) coefficients are expressed by the linear dynamic model in steady-state situation, and they are a function of parameters and the initial operating point of wind turbines.

V. Example

In an example with two identical wind turbines with $P_{s,1} = P_{s,2} = 0.1$, $h = 6.093$, and $\Omega_s = 10.62$, the dynamic equations (33) and (34) are

$$K\sin(\theta_0 + \delta\theta) - D(\delta s) = h\,\frac{d\delta s}{dt} \tag{35}$$

$$\frac{d\delta\theta}{dt} = -\Omega_s \delta s \tag{36}$$

where $\theta_0 = \theta_{2,1} - \theta_{1,0}$, $\delta\theta = \delta\theta_2 - \delta\theta_1$, $\delta s = \delta s_2 - \delta s_1$, $K = 8.55 \cdot 10^{-3}$, and $D = 121.42$.

Assuming $\sin(\delta\theta) \approx \delta\theta$ and $\cos(\delta\theta) \approx 1$, (35) and (36) can be expressed as

$$M\,\frac{d^2\delta\theta}{dt^2} = A + B\sin(\delta\theta) + C\,\frac{d(\delta\theta)}{dt} \tag{37}$$

where

$$A = K\sin(\theta_0) \quad B = K\cos(\theta_0) \quad C = \frac{D}{\Omega_s} \quad M = -\frac{h}{\Omega_s}.$$

The equation roots p_1 and p_2 of the differential equation can be expressed as

$$\begin{aligned}
p_1 &= \frac{C}{2M}\left(1 + \sqrt{1+\zeta}\right) \approx \frac{C}{M} = -19.92\\
p_2 &= \frac{C}{2M}\left(1 - \sqrt{1+\zeta}\right) \approx -\frac{K}{C} \ll p_1\\
\zeta &= \frac{4BM}{C^2} = -1.5 \cdot 10^{-4}\cos(\theta_0) \ll 1.
\end{aligned}$$

So the variable $\theta = \theta_0 + \delta\theta$, for $\theta_0 = 10^\circ$ and $p_2 = -7.4 \cdot 10^{-4}$, can be defined as $\theta = \theta_0 e^{p_2 t}$, the evolution of which is similar to that given in Fig. 1(a) obtained by simulation.

VI. Conclusions

In this paper, an analysis of synchronization conditions for a wind park with asynchronous wind turbines was developed. The synchronization phenomenon was simulated assuming mechanical sinusoidal power fluctuations. These fluctuations are assumed to simulate those produced by some known physical phenomena such as tower shadow, wind shear, and rotational sampling.

From the analysis, some conclusions can be extracted.

1) Synchronization is a phenomenon with a very slow time constant, which means that it is a very slow process. This is already observable in complex simulations.

2) Synchronization can be studied by means of simulation processes carried out with second-order differential equations for modeling the wind turbines. Several terms in the power expressions appear, some of which can be interpreted as synchronization and damping constants.
3) Synchronization only seems to be possible when the wind turbines are identical and operate under very similar conditions (mean values of operating points). The trend to synchronize means that, if these conditions are given and different wind turbines work at the same speed, then after along time, their blades will tend to pass in front of the towers at the same time.
4) As the wind power changes every few seconds in wind parks, a non synchronization process is expected there, except for partial synchronization in groups of machines close to each other. This seems to be a typical situation in a wind park.
5) The above-mentioned conclusions are of interest in the analysis of the effect of wind parks on perturbations in the electrical network, because the square root of n rule is under discussion for assessing such perturbations. The rule seems to be applicable, because synchronization of asynchronous wind turbines does not seem a probable process in general.

APPENDIX A
DATA

The data used in the simulations presented in the paper are as follows:

Base power = 350 kVA	$X_m = 2.78$ p.u.	$H = 3.025$ s
Base voltage = 690 V	$R_r = 0.006\,12$ p.u.	$\omega_s = 100 \cdot \pi$ rad/s
$X_r = 0.0639$ p.u.	$R_s = 0.005\,71$ p.u.	$V_{th} = 1$ p.u.
$X_s = 0.187\,81$ p.u.	$C = 1.1475 \cdot 10^{-3}$ p.u.	

APPENDIX B
TERMS OF THE LINEAR DYNAMIC MODEL OF A WIND PARK

The variation of stator current in the ith machine can be expressed as

$$\Delta \underline{I}_{s,i} = \frac{\Delta \underline{U}_i - \Delta \underline{E}'_i}{R_{s,i} + jX'_i} \tag{II.1}$$

where $\Delta \underline{U}_i$ is the nodal voltage defined by the nodal analysis of the circuit in Fig. 5, obtainable from (14).

Applying (16) and (19) to the system in Fig. 5, the following matrix expression is deduced:

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} A_{1,1} & \cdots & A_{1,2} & \cdots & A_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{i,1} & \cdots & A_{i,i} & \cdots & A_{1,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ A_{n,1} & \vdots & A_{n,i} & \cdots & A_{n,n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b_1 \\ \vdots \\ b_i \\ \vdots \\ b_n \end{bmatrix} \tag{II.2}$$

where

x_i — 3×1 vector of $[\Delta E'^r_i \;\; \Delta E'^m_i \;\; \Delta s_i]^t$ components;
$A_{i,i}$ — 3×3 matrix of parameters and initial conditions of machine i;
$A_{i,k}$ — 3×3 matrix that represents the relation between variables of machine i and machine k;
b_i — 3×1 vector defined as

$$\begin{bmatrix} 0 & 0 & \dfrac{\Delta P'_{m,i}}{h_i} \end{bmatrix}^t.$$

$$A_{i,i} = \begin{pmatrix} -\alpha_{i,i} & \omega_s s_{0,i} + \beta_{i,i} & \omega_s E'_{m0,i} \\ -\omega_s s_{0,i} - \beta_{i,i} & -\alpha_{i,i} & -\omega_s E'_{r0,i} \\ \dfrac{-c_{i,i}}{h_i} & \dfrac{-d_{i,i}}{h_i} & 0 \end{pmatrix}$$

$$A_{i,k} = \begin{pmatrix} \alpha'_{i,k} & -\beta'_{i,k} & 0 \\ \beta'_{i,k} & \alpha'_{i,k} & 0 \\ \dfrac{-c_{i,k}}{h_i} & \dfrac{-d_{i,k}}{h_i} & 0 \end{pmatrix}$$

$$\alpha_{i,i} + j\beta_{i,i} = \frac{1}{T'_{0,i}}\left(1 + \frac{j(X_{0,i} - X'_i)(1 - \underline{K}_{i,i})}{R_{s,i} + jX'_i}\right)$$

$$\alpha'_{i,k} + j\beta'_{i,k} = \frac{1}{T'_{0,i}} \frac{j(X_{0,i} - X'_i)\underline{K}_{i,k}}{R_{s,i} + jX'_i}$$

$$c_{i,i} = \text{Re}\{\underline{I}_{s,0,i}\} + G_{i,i}$$

$$d_{i,i} = \text{Im}\{\underline{I}_{s,0,i}\} + B_{i,i}$$

$$G_{i,i} + jB_{i,i} = \frac{\underline{E}_{i,0}(\underline{K}_{i,i} - 1)^*}{R_{s,i} - jX'_i}$$

$$c_{i,k} + jd_{i,k} = \frac{\underline{E}_{i,0}(\underline{K}_{i,k})^*}{R_{s,i} - jX'_i}$$

$$h_i = 2H_i(1 - s_{i,0.})$$

$\underline{K}_{i,i}$, $\underline{K}_{i,k}$ are elements of matrix K in (14).

APPENDIX C
MATRIX NOTATION

Equation (21) represents a set of equations with the shape shown in (III.1) and (III.2), shown at the bottom of the next page.

APPENDIX D
OTHER CONSTANTS USED

Taking into account only the constant values of (21), then (26)–(28) can be expressed as

$$\begin{bmatrix} \delta E'^r_i \\ \delta E'^m_i \end{bmatrix} \equiv \frac{\omega_s}{2}\begin{bmatrix} X^r_{(i+1)}X^r_{(i+2)} + X^m_{(i+1)}X^m_{(i+2)} \\ -X^r_i X^r_{(i+2)} - X^m_i X^m_{(i+2)} \end{bmatrix} \tag{IV.1}$$

$$[\delta P_{e,i}] = M\delta E + \text{Re}\left\{\frac{\underline{y}'_i}{2}\right\} \cdot \left((X^r_i)^2 + (X^m_i)^2 + (X^r_{i+1})^2 + (X^m_{i+1})^2\right) + \tfrac{1}{2}\left(X^r_i \sum_{k=1}^{m} \text{Re}\{\underline{y}'_k\} X^r_k\right.$$

$$
\begin{aligned}
&\quad +X^r_{i+1}\sum_{k=1}^{m}\mathrm{Re}\left\{\underline{y}'_k\right\}X^r_{k+1}\Bigg)\\
&+\tfrac{1}{2}\Bigg(X^m_i\sum_{k=1}^{m}\mathrm{Re}\left\{\underline{y}'_k\right\}X^m_k\\
&\quad +X^m_{i+1}\sum_{k=1}^{m}\mathrm{Re}\left\{\underline{y}'_k\right\}X^m_{k+1}\Bigg)\\
&+\tfrac{1}{2}\Bigg(X^r_{i+1}\sum_{k=1}^{m}\mathrm{Im}\left\{\underline{y}'_k\right\}X^r_k\\
&\quad -X^r_i\sum_{k=1}^{m}\mathrm{Im}\left\{\underline{y}'_k\right\}X^r_{k+1}\Bigg)\\
&+\tfrac{1}{2}\Bigg(X^m_{i+1}\sum_{k=1}^{m}\mathrm{Im}\left\{\underline{y}'_k\right\}X^m_k\\
&\quad -X^m_k\sum_{k=1}^{m}\mathrm{Im}\left\{\underline{y}'_k\right\}X^m_{k+1}\Bigg) \qquad \text{(IV.2)}
\end{aligned}
$$

$$
\delta P_{m,\,i}=\frac{\Omega_s}{2\Omega_0}\left(P_{s,\,i}\cos(\theta_{i,\,0})X^r_{i+2}+P_{s,\,i}\sin(\theta_{i,\,0})X^m_{i+2}\right). \qquad \text{(IV.3)}
$$

Furthermore, substituting in (IV.1) the values obtained in (21)

$$
\begin{bmatrix}\delta E'^r_i\\ \delta E'^m_i\end{bmatrix}=\begin{bmatrix}\displaystyle\sum_{k,\,n=1}^{m}\Lambda^{x,\,x}_{i,\,k,\,n}\cos(\theta_k-\theta_n)+\sum_{k,\,n=1}^{m}\Lambda^{x,\,y}_{i,\,k,\,n}\sin(\theta_k-\theta_n)\\ \displaystyle\sum_{k,\,n=1}^{m}\Lambda'^{x,\,x}_{i,\,k,\,n}\cos(\theta_k-\theta_n)+\sum_{k,\,n=1}^{m}\Lambda'^{x,\,y}_{i,\,k,\,n}\sin(\theta_k-\theta_n)\end{bmatrix} \qquad \text{(IV.4)}
$$

where

$$
\begin{aligned}
\Lambda^{x,\,x}_{i,\,k,\,n}&=\omega_s P_{s,\,k}P_{s,\,n}\cdot\left(D^r_{i+1,\,k+2}D^r_{i+2,\,n+2}+D^m_{i+1,\,k+2}D^m_{i+2,\,n+2}\right)\\
\Lambda^{x,\,y}_{i,\,k,\,n}&=\omega_s P_{s,\,k}P_{s,\,n}\cdot\left(D^r_{i+1,\,k+2}D^m_{i+2,\,n+2}-D^m_{i+1,\,k+2}D^r_{i+2,\,n+2}\right)\\
\Lambda'^{x,\,x}_{i,\,k,\,n}&=\omega_s P_{s,\,k}P_{s,\,n}\cdot\left(D^r_{i,\,k+2}D^r_{i+2,\,n+2}+D^m_{i,\,k+2}D^m_{i+2,\,n+2}\right)\\
\Lambda'^{x,\,y}_{i,\,k,\,n}&=\omega_s P_{s,\,k}P_{s,\,n}\cdot\left(D^r_{i,\,k+2}D^m_{i+2,\,n+2}-D^m_{i,\,k+2}D^r_{i+2,\,n+2}\right)\\
\Lambda^{x,\,x}_{i,\,i,\,n}&=\Lambda^{x,\,x}_{n,\,n,\,i}\qquad \Lambda^{x,\,y}_{i,\,i,\,n}=-\Lambda^{x,\,y}_{n,\,n,\,i}.
\end{aligned}
$$

Furthermore, for (IV.3)

$$
\delta P_{m,\,i}=\sum_{k=1}^{m}\Gamma^r_{i,\,k}\cos(\theta_{i,\,0}-\theta_{k,\,0})+\sum_{k=1}^{m}\Gamma^m_{i,\,k}\sin(\theta_{i,\,0}-\theta_{k,\,0}) \qquad \text{(IV.5)}
$$

where

$$
\Gamma^r_{i,\,k}=\frac{\Omega_s}{\Omega_0}P_{s,\,i}P_{s,\,k}D^r_{i+2,\,k+2}\qquad \Gamma^m_{i,\,k}=\frac{\Omega_s}{\Omega_0}P_{s,\,i}P_{s,\,k}D^m_{i+2,\,k+2}.
$$

Similarly, for electrical power (IV.2)

$$
\delta P_{e,\,i}=\sum_{k,\,n=1}^{m}\Pi^x_{i,\,k}\cos(\theta_{k,\,0}-\theta_{n,\,0})+\sum_{k,\,n=1}^{m}\cdot\Pi^y_{i,\,k}\sin(\theta_{k,\,0}-\theta_{n,\,0}) \qquad \text{(IV.6)}
$$

where the most important terms correspond to $M\delta E'$.

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$$
\begin{array}{c}
1\\ \vdots\\ i\\ \vdots
\end{array}
\begin{bmatrix}\begin{bmatrix}X_1\\X_{1+1}\\X_{1+2}\end{bmatrix}\\ \vdots\\ \begin{bmatrix}X_i\\X_{i+1}\\X_{i+2}\end{bmatrix}\\ \vdots\end{bmatrix}
=
\begin{bmatrix}
\begin{bmatrix}D_{1,1}&D_{1,1+1}&D_{1,1+2}\\D_{1+1,1}&D_{1+1,1+1}&D_{1+1,1+2}\\D_{1+2,1}&D_{1+2,1+1}&D_{1+2,1+2}\end{bmatrix}&\cdots&\begin{bmatrix}D_{1,i}&D_{1,i+1}&D_{1,i+2}\\D_{1+1,i}&D_{1+1,i+1}&D_{1+1,i+2}\\D_{1+2,i}&D_{1+2,i+1}&D_{1+2,i+2}\end{bmatrix}\\
\vdots&&\vdots\\
\begin{bmatrix}D_{i,1}&D_{i,1+1}&D_{i,1+2}\\D_{i+1,1}&D_{i+1,1+1}&D_{i+1,1+2}\\D_{i+2,1}&D_{i+2,1+1}&D_{i+2,1+2}\end{bmatrix}&\cdots&\begin{bmatrix}D_{i,i}&D_{i,i+1}&D_{i,i+2}\\D_{i+1,i}&D_{i+1,i+1}&D_{i+1,i+2}\\D_{i+2,i}&D_{i+2,i+1}&D_{i+2,i+2}\end{bmatrix}\\
\vdots&&\vdots
\end{bmatrix}
\blacksquare
\begin{bmatrix}\begin{bmatrix}0\\0\\\Delta\underline{P}_{1+2}\end{bmatrix}\\ \vdots\\ \begin{bmatrix}0\\0\\\Delta\underline{P}_{i+2}\end{bmatrix}\\ \vdots\end{bmatrix} \qquad \text{(III.1)}
$$

$$
\delta P_{e,i}=\begin{bmatrix}M_{i,1}&M_{i,1+1}&M_{i,1+2}\end{bmatrix}\cdots\begin{bmatrix}M_{i,i}&M_{i,i+1}&M_{i,i+2}\end{bmatrix}\cdots\blacksquare
\begin{bmatrix}\begin{bmatrix}\delta E'^r_1\\ \delta E'^m_1\\ \delta s_1\end{bmatrix}\\ \vdots\\ \begin{bmatrix}\delta E'^r_i\\ \delta E'^m_i\\ \delta s_i\end{bmatrix}\\ \vdots\end{bmatrix} \qquad \text{(III.2)}
$$

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