An Improved Branch-Exchange Algorithm for Large-Scale Distribution Network Planning

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***Abstract*—Design of optimal layout for medium-voltage power networks is a common issue in electrical distribution planning. Technical constraints (radial structure, voltage drops, and equipment capacity), and reliability limits must be fulfilled. The function for minimizing includes investments, power losses, and quality of supply costs. We present in this paper an improved algorithm based on a branch-exchange technique to solve large-scale problems. A heuristic algorithm for solving a Euclidean Steiner problem is used to improve the network by including transshipment nodes.**

***Index Terms*—Branch exchange, network design, power system distribution planning, Steiner tree problems.**

NOMENCLATURE

$N = N_C \cup N_S$	set of nodes;
N_C	set of load nodes;
N_S	set of substation nodes;
$E = \{(i,j)/i,j \in N\}$,	set of branches corresponding to distribution lines in which power flows from node i to j;
L_{ij}	length of the branch (i,j);
I_{ij}	current through the branch (i,j) that can be evaluated by $I_{ij} = \sum_{k \in D_j} I_k$;
I_k	current for the load power, e.g., for a three-phase load, it will be given by $I_k = S_k/(\sqrt{3}V)$, where S_k is the load (kVA), and V is the nominal voltage (kV);
L_j	$= L_{sk} + M_k \ / \ s \in N_S, k \in N_C, (s,k) \in P_{sj}$;
M_k	$= \sum_{(m,n) \in D_k} L_{mn}$;
$E_k = \{(i,j) \in E/i \in N_S,\ j \in N_C\}$	set of branches directly connected to a substation;
$D_j = \bigcup_{k \in N_C} P_{jk}$	subnetwork downstream from the node j (including j);
p_{jk}	path of nodes and branches $(j,(j,m),(m,\cdot)\ldots(\cdot,k),k)$;
K_I	coefficient of investment costs;
K_L	coefficient of cost of power losses;
K_R	coefficient depending on the reliability costs.

Manuscript received December 27, 1999; revised March 8, 2002. The authors wish to thank Unión Fenosa S.A. for supporting this work.

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Digital Object Identifier 10.1109/TPWRS.2002.804998

I. INTRODUCTION

INVESTMENTS in power distribution systems constitute a significant part of the utilities' expenses [1]. For this reason, efficient planning tools are needed to allow planners to reduce costs. Computer optimization algorithms improve cost reduction compared with systems designed by hand. So, in recent years, a lot of mathematical models and algorithms have been developed. A good review of classical models and issues and a recent overview can be found in [2] and [3], while [4] focuses on selection and application of optimization methods for distribution design.

A distribution system consists of a number of high-voltage/medium-voltage (HV/MV) substations and a radial network (loops are only possible for backup) fulfilling technical constraints (voltage drops and equipment capacity) and quality of supply limits, to feed the load demands.

The most common issue in distribution planning deals with the reinforcement of an existing system for growing load demand. However, occasionally, we need to design a system to meet load demand in new areas without existing facilities. This planning is often called "greenfield" planning [1], because the planner starts with nothing (a green field).

Over the years, heuristic branch-exchange algorithms have been developed in order to avoid the complexity and time consumption of practical large-scale optimization problems in distribution planning [5]–[10].

In this paper, we present an improved algorithm, based on a branch-exchange technique, to solve the problem that deals with obtaining optimal MV feeder layout and installed power in substations. The only known data are geographic location (x_i, y_i) of loads and substation and power consumption of customers. The cost function to be minimized consists of fixed and variable costs. Fixed costs are investments in MV networks, and variable costs correspond to power losses and reliability. Also, voltage-drop constraints and a minimum level of supply quality must be satisfied.

II. OPTIMIZATION PROBLEM

Mathematically, the "greenfield" optimization problem (Fig. 1) can be expressed by the obtaining of a set of trees

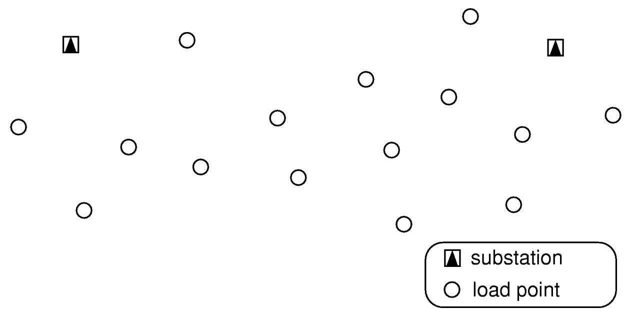

Fig. 1. Elements of greenfield planning.

(forest) $G(N,E)$ that minimizes a function Ψ' associated to the trees defined by

$$\Psi' = \sum_{(i,j)\in E} (K_I + K_L I_{ij}^2) L_{ij} + \sum_{(i,j)\in E_S} K_R I_{ij} L_j. \qquad (1)$$

Since the solution should be radial, for all $j \in N_C$ (load node) and for all $s \in N_S$, there is only one branch (i, j) arriving at j, and there are no branches (k, s) arriving at s.

Minimizing (1) is equivalent to minimizing

$$\Psi = \sum_{(i,j)\in E} (K + I_{ij}^2) L_{ij} + \sum_{(i,j)\in E_S} K_r I_{ij} L_j \qquad (2)$$

where $K = K_I/K_L$, $K_r = K_R/K_L$, and the optimal cost of the network will be $\min(\Psi') = K_L \cdot \min(\Psi)$.

On the other hand, the voltage constraint can be defined by

$$K_V \cdot \sum_{(m,n)\in P_{ij}} I_{mn} \cdot L_{mn} \le \Delta V^{\max}, \qquad \forall i \in N_S,\ j \in N_C \qquad (3)$$

where K_V depends on the conductor type, and $\Delta V^{\max}$ is the maximum admissible voltage drop.

Also, a reliability constraint, according to the number of annual outages, is established. This constraint can be written by

$$\Gamma_j = \lambda \cdot \Big(L_{sj} + \sum_{(m,n)\in D_j} L_{m,n}\Big) \le \Gamma^{\max} \qquad \forall\,(s,j) \in E_S \qquad (4)$$

where (s, j) is the branch arriving at j from the substation s; $\Gamma^{\max}$ is the maximum annual outage rate admissible by customers; and λ is the annual outage rate per length unit of line.

The reliability calculus are made only taking into account the circuit breaker placed in each outgoing line from the substation. The location of sectionalizers or any other switch in strategic locations downstream from the breaker is not considered.

III. Branch-Exchange Algorithm

The proposed algorithm is based on a branch-exchange technique, and begins with specified radial trees, called initial trees, that fulfill the constraints (3) and (4). These initial trees are made up connecting each load node to the nearest substation (Fig. 2), and these trees are the best solution, taking into account losses and reliability criteria. Basically, the branch-exchange algorithms consist of selecting a branch (i, j) and substituting it by another branch (m, n), where $n \in D_j$ and $m \in \overline{D}_j$, that produces a lower value for cost function (2), fulfilling constraints

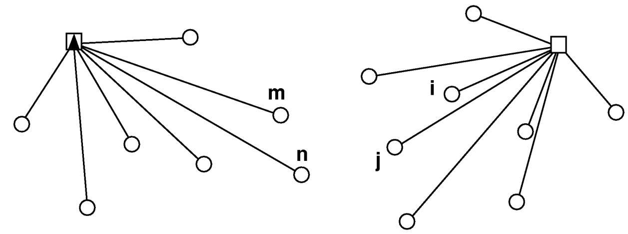

Fig. 2. Initial solution.

(3) and (4) as shown in Figs. 6 and 7. $\overline{D}_j$ is the complement of D_j in $G(N,E)$. This algorithm is described in Algorithm 1.

```
Algorithm 1 Branch-Exchange Algorithm
select an initial feasible solution
all branches become nonmarked (marks ← 0)
while local minimum exists do
 while nonmarked branches exists do
  select a nonmarked branch
  eliminate the selected branch
  connect with the best possible option.
  if best option == selected branch then
   mark the selected branch
  else
   substitute the selected branch by best option
 delete
   delete marks
  end if
 end while
 eliminate local minimums
end while
```

Some of the outstanding aspects of the algorithm are the following.

1) The branch selected (i, j) to be broken from among unmarked branches is the one less frequently marked. That is to say, when a selected branch results in an unsuccessful option because it is not possible to find another branch that diminishes the cost function, that branch is marked, and a counter is increased. When the branch selected to change results in success, then all marks are deleted.
2) The number of possible alternatives (m, n) depends on the selected branch (i, j) and takes values between $(k - 1)$ and $(k/2)^2$, where k is the number of nodes. Since this number could be very large, in order to avoid a time-consuming algorithm, a method to eliminate bad options and reduce the search area was developed.
3) As a consequence of the fact that only a pair of branches are exchanged simultaneously, the algorithm could obtain local minimums. In order to avoid this problem, a heuristic algorithm was developed that detects a crossing of branches. In this case, the crossing is eliminated by changing both branches at the same time (Fig. 3).

Once the solution is obtained by the process described before, a heuristic procedure is used to improve the network by adding new trans-shipment nodes (or Steiner nodes).

Thanks to the proposed techniques, this algorithm can be used for large network planning.

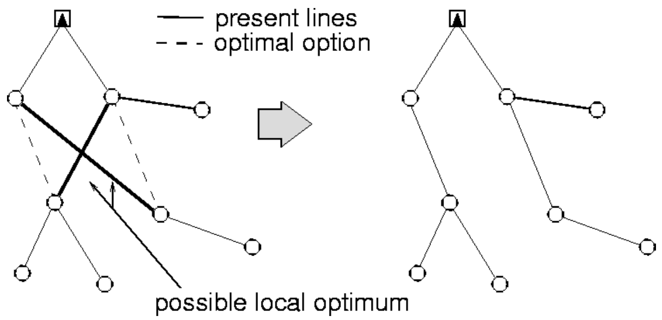

Fig. 3. Removal of a local minimum.

IV. Branch-Exchange Process

The cost variation $\Delta\Psi_{ij,nm}$ produced by the process of changing the branch (i,j) for the branch (m,n) is given by the following expression:

$$\Delta\Psi_{ij,mn} = \Phi_{ij,n} + A + K \cdot L_{mn} + B + C \quad (5)$$

where

$$\Phi_{ij,n} = I_{ij}^2 U_n - 2I_{ij}W_n - KL_{ij} \quad (6)$$

$$A = [2I_{ij}\bar{W}_m + I_{ij}^2 U_m] + I_{ij}^2 \cdot L_{mn} \quad (7)$$

$$B = K_r \cdot (-I_j^S \cdot (M_j + L_{ij}) - I_{ij} \cdot (L_j - M_j - L_{ij})) \quad (8)$$

$$C = K_r \cdot (\bar{I}_m^S \cdot (M_j + L_{mn}) + I_{ij} \cdot (\bar{L}_m + M_j + L_{mn})) \quad (9)$$

$$I_j^S = \frac{I_{sk}}{i \in N_S}, \; (s,k) \in P_{sj} \quad (10)$$

$$U_n = \sum_{(p,q)\in P_n} L_{pq} \quad (11)$$

$$W_n = \sum_{(p,q)\in P_n} I_{pq}L_{pq} \quad (12)$$

and $\bar{L}_m$, $\bar{I}_m^S$, and $\bar{W}_m$ are similar expressions to T_m, I_m^S, and W_m but are evaluated if the branch (i,j) is already disconnected and the value of the current though the upstream branches $((p,q) \in P_i)$ is modified.

The cost variation can be evaluated rapidly by the expression (5) if parameters L_i, U_i, M_i, and W_i are previously calculated and stored in every node i. As the main time-consuming process of the algorithm deals with searching new alternative branches that improve the cost function, the use of this expression becomes advantageous because those parameters only need to be calculated when a successful branch exchange has been carried out.

An alternative connection (m,n) will be accepted when the cost variation is negative ($\Delta\Psi_{ij,mn} < 0$), that is to say, when the cost decreases fulfilling the constraints. Therefore, we look for a branch (m,n) such that

$$\Phi_{ij,n} + A + K \cdot L_{mn} + B + C < 0. \quad (13)$$

Given that the rise of power losses and the cost of reliability as a result of connecting the set D_j to every node is always greater or equal to connecting it to the nearest substation, it follows that

$$A \geq I_{ij}^2 \cdot L_{n,\text{sub}} \quad (14)$$

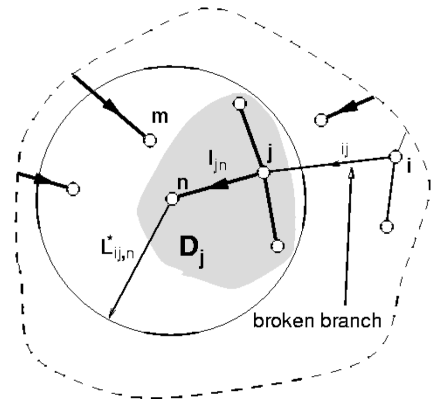

Fig. 4. Critical search radius.

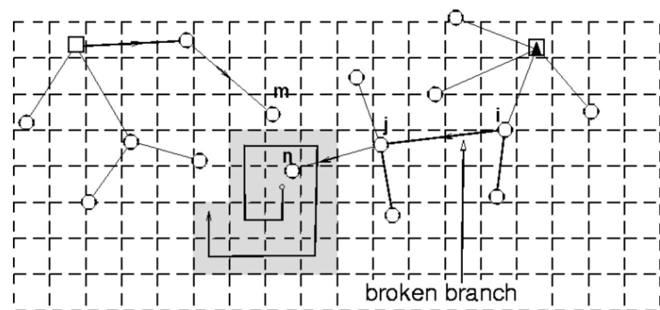

Fig. 5. Map of squares.

$$C \geq K_f \cdot I_{ij}(L_{n,\text{sub}} + M_j) \quad (15)$$

where $L_{n,\text{sub}}$ is the distance from the node n to the nearest substation. Then,

$$\Phi_{ij,n} \geq \Phi_{ij,n} + I_{ij}^2 \cdot L_{n,\text{sub}} + K \cdot L_{mn} + B + K_f \cdot I_{ij}(L_{n,\text{sub}} + M_j) \quad (16)$$

so that $\Phi_{ij,n} \geq 0$, and therefore, there is not any branch (m,n) that improves the cost function if $L_{mm} \geq L_{ij,n}$ where

$$L_{ij,n} = \frac{-1}{K}(\Phi_{ij,n} + I_{ij}^2 \cdot L_{n,\text{sub}} + B + K_f \cdot I_{ij}(L_{n,\text{sub}} + M_j)). \quad (17)$$

The search area is limited to the circle centered on node n and radius $L_{ij,n}$ (Fig. 4).

After the branch (m,n) has been found, so that $n \in D_j$ and $m \in \overline{D}_j$, and a cost reduction has been obtained ($\Delta\Psi_{ij,mn} < 0$), it is possible to reduce the search area. The new upper limit $L_{ij,n}^*$ is given by

$$L_{ij,n}^* = \frac{1}{K}(\Delta\Psi_{ij,mn} - \Phi_{ij,n} - I_{ij}^2 \cdot L_{n,\text{sub}} - B - K_f \cdot I_{ij}(L_{n,\text{sub}} + M_j)). \quad (18)$$

This may be written

$$L_{ij,n}^* = L_{ij,n} + \frac{\Delta\Psi_{ij,mn}}{K}. \quad (19)$$

Thus, there is not any branch (x,n) with $L_{xn} \geq L_{ij,n}^*$ so that $\Delta\Psi_{ij,xn} < \Delta\Psi_{ij,mn}$. Besides, $L_{ij,n}^* < L_{ij}$ because $\Delta\Psi_{ij,mn} < 0$. In this way, the search area decreases progressively as new alternative branches are found.

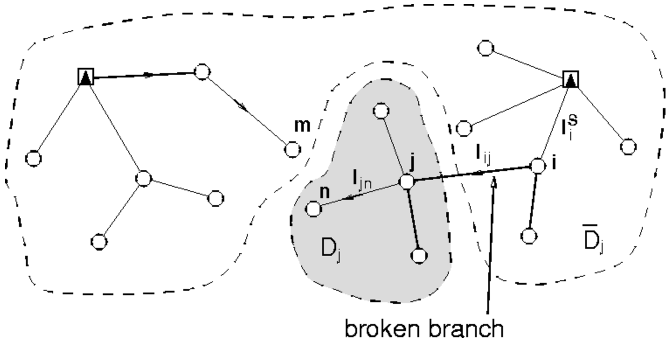

Fig. 6. Splitting the network.

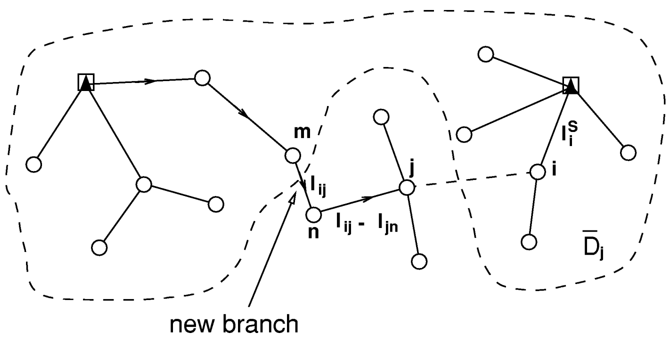

Fig. 7. Reconnecting the network.

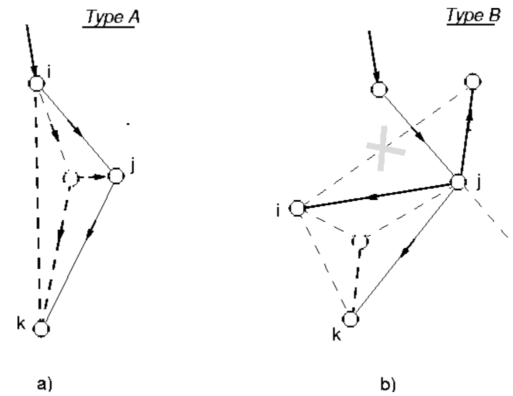

Fig. 8. Triangular structures.

A specific data structure is used to search new alternative branches, which consist of a matrix of lists. The whole area is divided into squares, and every square is associated with a list containing the nodes inside the square (Fig. 5). In this way, it is possible to search only the nodes inside the area limited by the length L^*_{xn} without evaluating the distance between node n and every node in $\overline{D}_j$. The order of search that is shown in Fig. 5, progresses until the upper limit L_{xn} falls down into the searched area. Using this method, only a small number of nodes are inspected, and the process can be run very fast.

V. Using the Theory of Steiner Nodes

The problem of designing a distribution network is quite similar to the well-known *Steiner tree problem* [11], which is concerned with finding the shortest network (in the Euclidean sense), connecting a set of points, and enabling the addition of auxiliary points. These points are called *Steiner points* [12]. However, in our case, we want to minimize the cost of the whole network, which not only depends on the length (Euclidean) of lines, but also on the power (current) that flows through the lines. The points to be connected are substations and transformers, and the auxiliaries are trans-shipment nodes.

Due to the problem being NP hard, heuristic algorithms have been developed to solve it. Most of them consist of systematically inserting new points from an initial spanning tree [13].

The heuristic algorithm proposed in this paper does the same. Starting with the optimal spanning tree obtained with the algorithm described in Section IV, it includes new trans-shipment nodes and looks for the optimal network.

A. Making Steiner Nodes

The process of making Steiner nodes is the following.

Step 1) All triangles (i, j, k) that fulfill the following conditions are identified and stored in a list.
- a) The branches (i, j) and (j, k), or in other cases, the branches (j, i) and (j, k), must belong to the tree. In the first case, it is stated that the triangle is *type A* and in the second case *type B* (Fig. 8).
- b) There is not any additional branch (j, q) that cuts the triangle (i, j, k).
- c) There is not any additional node inside the triangle.
- d) The interior angle made up by the sides of the triangle that belong to the tree is less than a previously established value.
- e) The node j is not a Steiner node.

Step 2) A new Steiner node s is created in the centroid of every triangle (i, j, k) previously selected.

Step 3) Branches belonging to the triangle (i, j, k) are eliminated from the network, and they are substituted by others that connect the vertex of the triangle to the new Steiner node. Two cases are possible.
- a) If the triangle is type A, then the additional branches are (i, s), (s, j) and (s, k).
- b) If the triangle is type B, the new branches are (j, s), (s, i), and (s, k).

Step 4) The previously selected triangle is retired from the list as are all of those triangles that share branches with the first one.

Step 5) All triangles that have a Steiner node as a vertex and have the characteristics that are required in Step 1) are added in the list.

Step 6) The previously described process is repeated from Step 2) if the list of triangles is not yet empty.

B. Optimal Location of the Trans-Shipment (Steiner) Nodes

After the Steiner nodes have been included and their connectivity with the remaining network nodes determined, the optimal geographic location of every Steiner node is calculated.

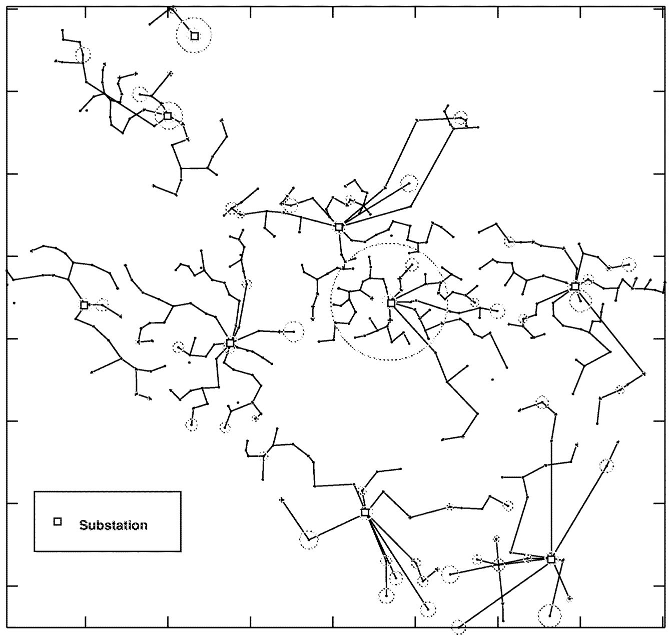

Fig. 9. Result of the Branch-Exchange Algorithm.

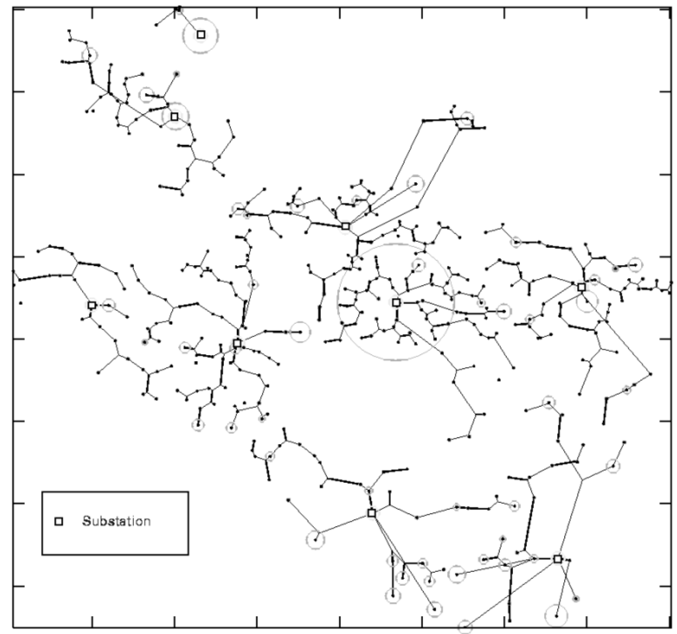

Fig. 10. Result of the heuristic Steiner algorithm.

Since the connectivity is defined, then the current through every branch is independent of the position of the Steiner points. Therefore, the cost of a branch is given by

$$\Psi_{ij} = (K + I_{ij}^2 + K_r \cdot I_j^S) \cdot L_{ij} = K_{ij} \cdot L_{ij}. \tag{20}$$

And the overall cost can be expressed by

$$\Psi = \sum_{(i,j)\in E_c} K_{ij}L_{ij} + \sum_{(i,j)\notin E_c} K_{ij}L_{ij} = \sum_{(i,j)\in E_c} K_{ij}L_{ij} + A \tag{21}$$

where

C set of trans-shipment nodes that have been included;

E $\{(i,j) \in E / i \in C$ or $j \in C\}$;

A constant, independent of the position of trans-shipment nodes.

If the length of a line L_{ij} is expressed as a function of the coordinates of the line ends, then (21) becomes

$$\Psi = \Psi(x_{c1}, \ldots, x_{ch}, y_{c1}, \ldots, y_{ch}) \tag{22}$$

where (x_{ci}, y_{ci}) are the coordinates of the Steiner node c_i, and h is the number of Steiner nodes.

Taking partial derivatives with respect to every variable and making the result equal to zero, the following expressions are obtained:

$$\frac{\partial\Psi}{\partial x_i} = \sum K_{ki} \cdot \left(\frac{x_k}{L_{ki}} - \frac{x_i}{L_{ki}}\right) = 0 \tag{23}$$

$$\frac{\partial\Psi}{\partial y_i} = \sum K_{ki} \cdot \left(\frac{y_k}{L_{ki}} - \frac{y_i}{L_{ki}}\right) = 0 \tag{24}$$

where $i \in C$.

The number of equations obtained is $2h$; consequently, the number of equations equals the number of unknown variables (x_{ci} and y_{ci} coordinates), and the problem can be solved by a classical method for nonlinear equations: Newton–Raphson, Gauss, Gauss–Seidel, etc. In this paper, for simplicity, the Gauss–Seidel method was used as described below.

Equations (23) and (24) can be written in an implicit form as follows:

$$x_i = \frac{\sum_{k\in H(i)} \frac{K_{ki}\cdot x_k}{L_{ki}}}{\sum_{k\in H(i)} \frac{K_{ki}}{L_{ki}}} \quad y_i = \frac{\sum_{k\in H(i)} \frac{(K_{ki}\cdot y_k)}{L_{ki}}}{\sum_{k\in H(i)} \frac{K_{ki}}{L_{ki}}}. \tag{25}$$

The iterative procedure of calculus is the following:

```
repeat
 Determine L_ki and set the more recent
   values of x_k e y_k
 Calculate the new values of x_i e y_i.
until ∑[x_i^m − x_i^{m−1}]^2 + [y_i^m − y_i^{m−1}]^2 > ε
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In the above procedure, x_i^m and y_i^{m-1} are the coordinates corresponding to the iteration $m-1$, (x_i^m, y_i^m) the ones for iteration m; and ε is the admissible error.

VI. Results

In order to show the capability of the algorithm for solving large-scale problems in a short period of time, the following example is presented. It consists of 387 load nodes and nine distribution substations located in an area of 5925 km^2 (79 km long and 75 km wide).

Fig. 9 shows the solution obtained with the Branch-Exchange Algorithm presented in Section IV. This solution is improved by using the algorithm to include new Steiner nodes described in Section V, and the result shown in Fig. 10 was obtained.

The cost function to be minimized is defined by (1) where $K_I = 5.20$ MPts/km, $K_L = 0.000\,162$ MPts/km $\cdot$ A^2, and $K_R = 0.012\,356$ MPts/km $\cdot$ A(1 MPts $= 5440$ USD).

On the other hand, the constraints established by (3) and (4) have been considered, taking into account a value of the maximum admissible voltage drop equal to 7% and an annual outage rate $\lambda = 0.15$ outages/year · km.

The minimum value for the cost function obtained is 5647 MPts (921 km of lines).

The algorithm was executed using an IBM RS-6000 43P workstation. The execution time was 3 s for the Branch-Exchange Algorithm and less than 1 s for the Steiner nodes algorithm.

VII. Conclusion

A branch-exchange technique and a methodology derived from heuristic Euclidean Steiner trees algorithm was developed and presented in this paper for an optimal design of MV networks. It permits us to obtain an acceptable solution for large-scale problems (thousand nodes), which would be impossible to solve using mathematical methods. The results were satisfactory enough for planning proposes.

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